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Robust output feedback H_∞ control of uncertain Markovian jump systems with mode-dependent time-delays

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This paper describes the synthesis of robust output feedback H_∞ control for a class of uncertain Markovian jump linear systems with time-delays which are time-varying and depend on the system modes of operation. Under the assumption of known bounds of system uncertainties and the control system gain variations, we present sufficient conditions on the existence of robust stochastic stability and γ-disturbance H_∞ attenuation. Through the changes of variables and Schur complements, these sufficient conditions can be rewritten in a set of coupled linear matrix inequalities, with which, robust control can be easily constructed. As an added advantage, the control design depends only on $p^o_{mj,t}$, the measured parameter of Markovian jumping at time t, which might be corrupted by measurement noises. Numerical examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

In recent years, much attention has been devoted to the time-delayed uncertain jump linear systems with Markovian jumping parameters (Benjelloun and Boukas 1997, 1998, Boukas and Liu 2000, 2001, Wang *et al*. 2002, 2003, Yuan *et al*. 2003, Chen *et al*. 2003, 2004b and Yuan and Mao 2004). The system is described by a set of time-delayed linear systems with the transitions between models determined by a Markov chain in a finite mode set. With the maturity of H_∞ control theory, many works have been devoted to H_∞ control of time-delayed uncertain jump linear systems under the assumptions of (i) known jumping parameter variations, and (ii) precise implementation of control systems. In Benjelloun *et al*. (1999), sufficient conditions were given on the existence of robust stochastic stabilization, and in Cao and James (2000), γ-suboptimal H_∞ state-feedback control was presented using the stochastic Lyapunov functional. Stabilization and H_∞ control via memoryless state feedback were introduced in Chen *et al*. (2004a) for discrete-time jump linear systems with mode-depended time delays using bounding technique for the cross terms. In Xu *et al*. (2003), H_∞ filtering was designed to ensure robust exponential mean-square stability of the filtering error system, and the $\mathcal{L}_2$-induced gain from the noise signal to the estimation error was bounded by a prescribed value.

For unknown jumping parameters, adaptive stabilization has been studied (Caines and Chen 1985, Chen and Caines 1989, Nassiri-Toussi and Caines 1991), where the system state can be observed, but the jumping parameter process cannot be directly measured and is estimated by the Wonham filter under known bounds of the system matrices. In Caines and Zhang (1995), the existence was established for adaptive feedback control of jump systems. The proposed adaptive certainty equivalence feedback control, which employs parameter estimates generated by the non-linear filter, stabilizes the system in an average mean square sense without any restrictions on bounds of the system matrices. In Tan *et al*. (2005), a sampled-data based parameter estimator and linear quadratic adaptive control were proposed for the case where only the sampled-data rather than the complete

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process of the system state is available for control design in addition to the unmeasurable jumping parameter process, and the suboptimality of control occurs when the sample step size is small enough.

In practice, precise control implementation is not possible due to the finite word length in digital processing, and the imprecision inherent in analog systems. It is shown that robust control schemes, including H_∞, μ or l_1 synthesis techniques exhibit poor stability margins if not properly implemented (Keel and Bhattacharyya 1997). To make the system insensitive to gain variations, non-fragile control design has been investigated for deterministic uncertain systems (Famularo *et al.* 1998, Kim *et al.* 1999, Yang and Wang 2001).

The existence of time delays may degrade the control performance and make stabilization become more difficult, especially when the delays are not perfectly known. Using appropriate Lyapunov–Krasovskii functionals (Hale 1977), the uncertainties from unknown time delays can be compensated for, such that the design of the stabilizing control law is free from these uncertainties (Ge *et al.* 2002, 2005). To avoid possible conservativeness due to many conservative transformations or the derivation of the stability conditions, several types of techniques have been used to obtain less conservative robust results for uncertain delay-dependent deterministic systems (Park 1999, Moon *et al.* 2001, Fridman and Shaked 2002, 2003). In Wu *et al.* (2004) the relationship between the terms in the Leibniz–Newton formula have been discussed, which is useful in determine the optimal ones by solving the corresponding linear matrix inequalities for less conservative results.

In this paper, robust output feedback H_∞ control is investigated for a class of uncertain Markovian jump linear systems with time-delays which are time-varying and depend on the system modes of operation. We will consider two jumping parameter measurement errors, detection delays and false alarms, in a failure detection and identification of the Markovian jumping parameters, which are not possible to be suppressed regardless of the sensors and algorithms used (Mariton 1991). The main contributions in this paper lie in

(i) the sufficient conditions on the existence of robust stochastic stability and γ disturbance H_∞ attenuation established directly by the measured parameter of Markovian jumping;
(ii) robust control design to overcome multiplicative uncertainties of its gains; and
(iii) delay-dependent stability criteria to jump systems for less conservative stability results.

The rest of the paper is organized as follows. The problem formulation and some preliminaries are given in § 2. Sufficient conditions on the existence of robust output feedback H_∞ control are presented in § 3. A simulation example is given in § 4, and followed by § 5 which concludes the work.

2. Problem formulation and preliminaries

Consider the following class of uncertain linear stochastic systems with Markovian jumping parameters and mode-dependent time delays:

$$\begin{cases} \dot{x}(t) = \left[A_1(p_{mj,t}) + \Delta_{A_1}(p_{mj,t},t)\right]x(t) \\ \qquad\quad + \left[A_2(p_{mj,t}) + \Delta_{A_2}(p_{mj,t},t)\right]x(t-\tau_{p_{mj,t}}(t)) \\ \qquad\quad + \left[B_1(p_{mj,t}) + \Delta_{B_1}(p_{mj,t},t)\right]u(t) + B_2(p_{mj,t})w(t) \\ z(t) = \left[C(p_{mj,t}) + \Delta_C(p_{mj,t},t)\right]x(t) \\ x(s) = f(s), \quad p_{mj,s} = p_{mj,0}, \quad s \in [-2\mu, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{m_1}$, $z(t) \in \mathbb{R}^{m_3}$ are the system states, inputs and outputs, respectively; $f(t) \in \mathbb{R}^n$ is a continuous vector valued initial function; $w(t) \in \mathbb{R}^{m_2}$ is the exogenous disturbance signal that belongs to $L_2[0,\infty)$; $p_{mj,t}$ is the parameter of Markovian jumping at time t; $\tau_{p_{mj,t}}(t)$ is the time-varying delay; and real constant matrices $A_1(p_{mj,t}) \in \mathbb{R}^{n\times n}$, $A_2(p_{mj,t}) \in \mathbb{R}^{n\times n}$, $B_1(p_{mj,t}) \in \mathbb{R}^{n\times m_1}$, $B_2(p_{mj,t}) \in \mathbb{R}^{n\times m_2}$, $C(p_{mj,t}) \in \mathbb{R}^{m_3\times n}$ are known matrices representing the nominal system parameters, while the unknown matrices $\Delta_{A_1}(p_{mj,t},t) \in \mathbb{R}^{n\times n}$, $\Delta_{A_2}(p_{mj,t},t) \in \mathbb{R}^{n\times n}$, $\Delta_{B_1}(p_{mj,t},t) \in -\mathbb{R}^{n\times m_1}$, $\Delta_C(p_{mj,t},t) \in \mathbb{R}^{m_3\times n}$ represent time-varying parameter uncertainties due to the presence of environmental noises, disturbances, and modelling uncertainties (Benjelloun *et al.* 1999, Cao and James 2000, Chen *et al.* 2003). For the stochastic system under study, the following Assumptions are in order.

Assumption 1: *The Markovian jumping parameter, $p_{mj,t}$, is a right-continuous Markov process and takes values in finite set $S = \{1,2,\ldots,N\}$, defined by*

$$P\{p_{mj,t+\Delta} = j | p_{mj,t} = i\} = \begin{cases} q_{ij}\Delta + o(\Delta), & i \neq j \\ 1 + q_{ii}\Delta + o(\Delta), & i = j \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, q_{ij} is the transition rate from i to j, and

$$q_{ii} = -\sum_{j\neq i} q_{ij}, \quad q_{ij} \geq 0, \quad j \neq i. \tag{2}$$

Assumption 2: *The time-varying delay, $\tau_{p_{mj,t}}(t)$, satisfies*

$$0 < \tau_{p_{mj,t}}(t) \leq \mu_{p_{mj,t}} \leq \mu < \infty, \quad \dot{\tau}_{p_{mj,t}}(t) \leq h_{p_{mj,t}} < 1, \quad \forall p_{mj,t} \in S \tag{3}$$

with $\mu_{p_{mj,t}}$ and $h_{p_{mj,t}}$ being real constant scalars for each $p_{mj,t} \in S$, and $\mu = \max_{i\in S}\{\mu_i\}$.

Assumption 3: *The uncertain parameters are assumed to be of the form* (Xu 2003):

$$\begin{aligned}
\Delta_{A_1}(p_{mj,t},t) &= H_1(p_{mj,t})F(p_{mj,t},t)E_1(p_{mj,t})\\
\Delta_{A_2}(p_{mj,t},t) &= H_1(p_{mj,t})F(p_{mj,t},t)E_2(p_{mj,t})\\
\Delta_{B_1}(p_{mj,t},t) &= H_1(p_{mj,t})F(p_{mj,t},t)E_3(p_{mj,t})\\
\Delta_{C}(p_{mj,t},t) &= H_2(p_{mj,t})F(p_{mj,t},t)E_4(p_{mj,t}),
\end{aligned}$$

where $H_1(p_{mj,t})\in\mathbb{R}^{n\times n_f}$, $H_2(p_{mj,t})\in\mathbb{R}^{m_3\times n_f}$, $E_1(p_{mj,t})\in\mathbb{R}^{n_f\times n}$, $E_2(p_{mj,t})\in\mathbb{R}^{n_f\times n}$, $E_3(p_{mj,t})\in\mathbb{R}^{n_f\times m_1}$ *and* $E_4(p_{mj,t})\in\mathbb{R}^{n_f\times n}$ *are known real constant matrices, and* $F(p_{mj,t},t)\in\mathbb{R}^{n_f\times n_f}$ *are the uncertain time-varying matrices satisfying*

$$F^T(p_{mj,t},t)F(p_{mj,t},t)\le I,\quad \forall p_{mj,t}\in S. \tag{4}$$

Remark 1: The parameter uncertainty structure as in Assumption 3 is an extension of the so-called matching condition, which has been widely used in the problems of robust control and robust filtering of uncertain linear systems (see, e.g., Benjelloun and Boukas (1997), Benjelloun *et al.* (1999), Cao and James (2000), Boukas and Liu (2000, 2001), Chen *et al.* (2003, 2004a) and Xu *et al.* (2003)). The matrices $H_1(p_{mj,t})$, $H_2(p_{mj,t})$, $E_1(p_{mj,t})$, $E_2(p_{mj,t})$, $E_3(p_{mj,t})$ and $E_4(p_{mj,t})$ characterize how the uncertain parameters in $F(p_{mj,t},\ t)$ affects the nominal matrices $A_1(p_{mj,t})$, $A_2(p_{mj,t})$, $B_1(p_{mj,t})$ and $C(p_{mj,t})$ of system (1).

In practical control systems, due to the existence of environmental noises, disturbances, and small modelling uncertainties, non-zero detection delays and false alarms may occur in the mode detection of the plant. To describe such phenomena, two stochastic processes are needed, one is the $p_{mj,t}$ in (1), which describes the real jump process of the plant mode, and the other (we call it $p^0_{mj,t}$) describes the measurement of $p_{mj,t}$, which can be used for control design. For convenience of citation, as in Mariton (1990, 1991) the following models are assumed to describe the above two measurement errors:

- When $p_{mj,t}$ jumps from i to j, $p^o_{mj,t}$ is an independent exponentially distributed random variable with mean $1/q^0_{ij}$, governed by

$$P\left\{p^o_{mj,t+\Delta}=j\left|\begin{matrix}p^o_{mj,s}=i\\ p_{mj,t_0}=j\\ p_{mj,t_{0^-}}=i\\ s\in[t_0,t]\end{matrix}\right.\right\}=\begin{cases}q^0_{ij}\Delta+o(\Delta). & i\ne j\\ 1+q^0_{ij}\Delta+o(\Delta), & i=j.\end{cases} \tag{5}$$

The entries of the matrix, $[q^0_{ij}]\in\mathbb{R}^{N\times N}$, are evaluated from observed sample paths, and

$$q^0_{ii}=-\sum_{j\ne i}q^0_{ij},\quad (q^0_{ij}\ge 0,\ j\ne i). \tag{6}$$

- When $p_{mj,t}$ remains at i, $p^o_{mj,t}$ has transitioned from i to j occasionally. An independent exponential distribution with rate q^1_{ij} is assumed to be

$$P\left\{p^o_{mj,t+\Delta}=j\left|\begin{matrix}p^o_{mj,s}=i\\ s\in[t_0,\ t]\end{matrix}\right.\right\}=\begin{cases}q^1_{ij}\Delta+o(\Delta), & i\ne j\\ 1+q^1_{ii}\Delta+o(\Delta), & i=j\end{cases} \tag{7}$$

with a matrix, $[q^1_{ij}]\in\mathbb{R}^{N\times N}$, of false alarm rates, which can also be valued from observed sample paths, and

$$q^1_{ii}=-\sum_{j\ne i}q^1_{ij},\quad (q^1_{ij}\ge 0,\ j\ne i). \tag{8}$$

For simplicity, in the sequel, let $M_{ji}(t)$ denote the corresponding matrix, $M(p^o_{mj,t},p_{mj,t},t)$, for each $p^o_{mj,t}=j$, $p_{mj,t}=i,j,i\in S$, and we assume that the initial time is $t_0=0$, and the initial time values $x(0)=x_0$, $p_{mj,0}$ and $p^o_{mj,0}$ are deterministic.

In this paper, we are concerned with the design of the following linear output feedback control law:

$$\begin{cases}\dot{\hat{x}}(t)=A_3(p^o_{mj,t})\hat{x}(t)+B_3(p^o_{mj,t})z(t)\\ u(t)=K(p^o_{mj,t})\hat{x}(t),\end{cases} \tag{9}$$

where $\hat{x}(t)\in\mathbb{R}^n$ is the state of the control, and $A_3(p^o_{mj,t})$, $B_3(p^o_{mj,t})$, $K(p^o_{mj,t})$ are design parameter matrices with appropriate dimensions to be determined.

In practice, precise control implementation is not possible. In this paper, we assume that the imprecision is described by additive errors, $\alpha(p_{mj,t})\phi(p_{mj,t},t)$, of the control gains, then the actual control is given by

$$u(t)=\left[I+\alpha(p_{mj,t})\phi(p_{mj,t},t)\right]K(p^o_{mj,t})\hat{x}(t), \tag{10}$$

where $\alpha(p_{mj,\ t})$ is positive constant for each $p_{mj,t}\in S$ and $\phi(p_{mj,t},t)$ is defined as

$$\phi^T(p_{mj,t},t)\phi(p_{mj,t},t)\le I,\quad \forall p_{mj,t}\in S.$$

Remark 2: Note that the control design only depends on the measured parameter, $p^o_{mj,t}$ of Markovian jumping, while the dynamic system is driven by the actual mode $p_{mj,t}$, that is the reconfiguration of the stabilization law consists of a switch of its gain $K(p^o_{mj,t})$ based on $p^o_{mj,t}$. The term $\alpha(p_{mj,t})\phi(p_{mj,t},t)$ represents the inherent variations of the control gains in the actual mode $p_{mj,t}$ at time t and has nothing to do with $p^o_{mj,t}$.

Applying the control (9) to system (1), letting $\xi(t)=[x^T(t),\hat{x}^T(t)]^T$, we obtain the closed-loop system

$$\begin{cases}\dot{\xi}(t)=\bar{A}_1(p^o_{mj,t},p_{mj,t},t)\xi(t)+\bar{A}_2(p_{mj,t})I_0\xi(t-\tau_{p_{mj,t}}(t))\\ \qquad +\bar{B}_2(p_{mj,t})w(t)\\ z(t)=\left[C(p_{mj,t})+\Delta_C(p_{mj,t},t)\right]I_0\xi(t)\\ I_0\xi(s)=f(s),\quad p_{mj,s}=p_{mj,0},\quad s\in[-2\mu,0],\end{cases} \tag{11}$$

where

$$\bar{A}_{1ji} = \begin{bmatrix} A_{1i} + \Delta_{A_{1i}}(t) & (B_{1i} + \Delta_{B_{1i}}(t))(I + \alpha_i\phi_i(t))K_j \\ B_{3j}(C_i + \Delta_{C_i}(t)) & A_{3j} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$$

$$\bar{A}_{2t} = \begin{bmatrix} A_{2i} + \Delta_{A_{2i}}(t) \\ 0 \end{bmatrix} \in \mathbb{R}^{2n\times n}, \quad \bar{B}_{2i} = \begin{bmatrix} B_{2i} \\ 0 \end{bmatrix} \in \mathbb{R}^{2n\times m_2},$$

$$I_0 = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n\times 2n}$$

for each $p^o_{mj,t} = j, p_{mj,t} = i, \forall i, j \in \mathcal{S}$.

Our control design objectives are as follows:

(i) *robust stabilization*. Determine the nominal control gain $K(p^o_{mj,t})$ in (10) and establish sufficient conditions for the system (1) such that the resulting closed-loop system (11) is robustly exponentially stable in mean square; and

(ii) H_∞ *control problem*. Given a constant scalar $\gamma > 0$, determine the nominal control gain $K(p^o_{mj,t})$ in (10) and establish the sufficient conditions such that the resulting closed-loop system (11) is robustly stochastically stable with disturbance attenuation level γ under zero initial condition ($x(0) = 0$), that is

$$J = E\left\{\int_0^T \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right]\mathrm{d}t\right\} < 0, \quad \forall w(t) \neq 0,$$

$$w(t) \in L_2[0, \infty). \tag{12}$$

3. Robust control

In this section, we consider the robust stability of the time-delayed uncertain jump linear closed system (11) without disturbance. To prove the main results, the following lemmas are required.

Lemma 1 (Kreindler and Jameson 1972) (Schur complement)**:** *Consider the following matrix of appropriate dimension*

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}, \quad Q_{22} > 0. \tag{13}$$

Then, Q *is positive definite if and only if* $Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T > 0$.

Lemma 2 (Xie 1996)**:** *Given matrices* $Q = Q^T$, H, E *and* $R = R^T > 0$ *of appropriate dimensions, then*

$$Q + HFE + E^T F^T H^T < 0$$

for all F *satisfying* $F^T F \le R$, *if and only if there exits some* $\rho > 0$ *such that*

$$Q + \rho HH^T + \rho^{-1} E^T RE < 0.$$

Lemma 3 (Mariton 1990) (Infinitesimal generator, $\mathscr{L}$, of Markov process)**:** *For the jump system*

$$\dot{x}(t) = f(x(t), u(t), p^o_{mj,t}, p_{mj,t}, t),$$

assume that $f(\cdot)$ *is continuous for all its variables within their domain of definition, and satisfies the usual growth and smoothness hypothesis,* $g(x(t), p^o_{mj,t}, p_{mj,t}, t)$ *is a scalar continuous function of* t *and* $x(t)$ *for each* $p^o_{mj,t}, p_{mj,t} \in S$. *Then, the infinitesimal generator,* $\mathscr{L}$, *of the random process* $\{x(t), p^o_{mj,t}, p_{mj,t}, t\}$ *can be described as follows.*

- *When* $p^o_{mj,t} = p_{mj,t} = i$, *we have*

$$\begin{aligned}\mathscr{L}g(x(t),i,i,t) &= \lim_{\Delta\to 0}\frac{1}{\Delta}\Big[E\Big\{g(x(t+\Delta), p^o_{mj,t+\Delta}, p_{mj,t+\Delta}, t+\Delta) \\ &\quad |x(t) = x, p^o_{mj,t} = i, p_{mj,t} = i, t\Big\} - g(x,i,i,t)\Big] \\ &= g_t(x,i,i,t) + f^T(x,u(t),i,i,t)g_x(x,i,i,t) \\ &\quad + \sum_{j=1}^N q_{ij}g(x,i,j,t) + \sum_{j=1}^N q^1_{ij}g(x,j,i,t).\end{aligned}$$

- *When* $p^o_{mj,t} = j \neq p_{mj,t} = i$, *we have*

$$\begin{aligned}\mathscr{L}g(x(t),j,i,t) &= \lim_{\Delta\to 0}\frac{1}{\Delta}\Big[E\Big\{g(x(t+\Delta), p^o_{mj,t+\Delta}, p_{mj,t+\Delta}, t+\Delta) \\ &\quad |x(t) = x, p^o_{mj,t} = j, p_{mj,t} = i, t\Big\} - g(x,j,i,t)\Big] \\ &= g_t(x,j,i,t) + f^T(x,u(t),j,i,t)g_x(x,j,i,t) \\ &\quad + q^0_{ji}g(x,i,i,t) - q^0_{ji}g(x,j,i,t).\end{aligned}$$

Theorem 1: *The uncertain time-delayed jump linear system* (11) *without disturbance* (*i.e.,* $w(t) \equiv 0$) *achieves exponential mean-square stability, and the output feedback control law* (9) *is robust if there are positive-definite matrices* $P_{ji} = P_{ji}^T > 0$, $Q = Q^T > 0$, $Z = Z^T > 0$, *positive semi-definite matrices* $X_{ji} \ge 0$, *constants* $\rho_{1ji} > 0$, $\rho_{2ji} > 0$,

$\rho_{3ji} > 0$, *and appropriately dimensioned matrices* K_j, Y_{ji}, T_{ji} *such that*

$$\bar{\mathbf{W}}_{ji} = \begin{bmatrix} L_1 & P_{1ji}B_{1i} + \rho_{3ji}E_{1i}^T E_{3i} & 0 & P_{1ji}H_{1i} & 0 & K_j^T \\ L_2 & 0 & K_j^T & 0 & V_{ji}^T H_{2i} & 0 \\ L_3 & \rho_{3ji}E_{2i}^T E_{3i} & 0 & 0 & 0 & 0 \\ L_4 & \mu Z B_{1i} & 0 & \mu Z H_{1i} & 0 & 0 \\ L_5 & -I + \rho_{3ji}E_{3i}^T E_{3i} & 0 & 0 & 0 & 0 \\ L_6 & 0 & -I + \rho_{1ji}\alpha_i^2 I & 0 & 0 & 0 \\ L_7 & 0 & 0 & -\rho_{3ji}I & 0 & 0 \\ L_8 & 0 & 0 & 0 & -\rho_{2ji}I & 0 \\ L_9 & 0 & 0 & 0 & 0 & -\rho_{1ji}I \end{bmatrix} < 0 \tag{14}$$

$$\Gamma_{ji} = \begin{bmatrix} X_{1ji} & X_{2ji} & I_0^T Y_{ji} \\ X_{2ji}^T & X_{3ji} & T_{ji} \\ Y_{ji}^T I_0 & T_{ji}^T & Z \end{bmatrix} \geq 0, \quad \forall i, j \in S, \tag{15}$$

where

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \\ L_9 \end{bmatrix} = \begin{bmatrix} \Phi_{11} + \rho_{3ji}E_{1i}^T E_{1i} + \rho_{2ji}E_{4i}^T E_{4i} & C_i^T V_{ji} + \mu X_{1ji}^2 & \Phi_{13} + \rho_{3ji}E_{1i}^T E_{2i} & \mu A_{1i}^T Z \\ V_{ji}^T C_i + \mu X_{1ji}^{2^T} & U_{ji} + U_{ji}^T + \Phi_{22} & \mu X_{2ji}^2 & 0 \\ \Phi_{13}^T + \rho_{3ji}E_{2i}^T E_{1i} & \mu X_{2ji}^{2^T} & \Phi_{33} + \rho_{3ji}E_{2i}^T E_{2i} & \mu A_{2i}^T Z \\ \mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z \\ B_{1i}^T P_{1ji} + \rho_{3ji}E_{3i}^T E_{1i} & 0 & \rho_{3ji}E_{3i}^T E_{2i} & \mu B_{1i}^T Z \\ 0 & K_j & 0 & 0 \\ H_{1i}^T P_{1ji} & 0 & 0 & \mu H_{1i}^T Z \\ 0 & H_{2i}^T V_{ji} & 0 & 0 \\ K_j & 0 & 0 & 0 \end{bmatrix}$$

$$X_{ji} = \begin{bmatrix} X_{1ji} & X_{2ji} \\ X_{2ji}^T & X_{3ji} \end{bmatrix} = \begin{bmatrix} X_{1ji}^1 & X_{1ji}^2 & X_{2ji}^1 \\ X_{1ji}^{2^T} & X_{1ji}^3 & X_{2ji}^2 \\ X_{2ji}^{1^T} & X_{2ji}^{2^T} & X_{3ji} \end{bmatrix}$$

with

$$\Phi_{11} = \begin{cases} A_{1i}^T P_{1ii} + P_{1ii}A_{1i} + \sum_{j=1}^{N} q_{ij}P_{1ij} + \sum_{j=1}^{N} q_{ij}^1 P_{1ji} + Y_{ii} + Y_{ii}^T + (1+\eta\mu)Q + \mu X_{1ii}^1, & \text{if } j = i \\ A_{1i}^T P_{1ji} + P_{1ji}A_{1i} + q_{ji}^0(P_{1ii} - P_{1ji}) + Y_{ji} + Y_{ji}^T + (1+\eta\mu)Q + \mu X_{1ji}^1, & \text{if } j \neq i \end{cases}$$

$$\Phi_{22} = \begin{cases} \sum_{j=1}^{N} q_{ij}P_{2ij} + \sum_{j=1}^{N} q_{ij}^1 P_{2ji} + \mu X_{1ii}^3, & \text{if } j = i \\ q_{ji}^0(P_{2ii} - P_{2ji}) + \mu X_{1ji}^3, & \text{if } j \neq i \end{cases} \tag{16}$$

$$\Phi_{13} = P_{1ji}A_{2i} - Y_{ji} + T_{ji}^T + \mu X_{2ji}^1, \quad \Phi_{33} = -T_{ji} - T_{ji}^T - (1-h_i)Q + \mu X_{3ji}$$

$$\eta = \max_{i \in S}\{|q_{ii}|\}, \quad V_{ji} = B_{3j}^T P_{2ji}, \quad U_{ji} = A_{3j}^T P_{2ji}.$$

The proof of this theorem is put into Appendix A.

Remark 3: It can be seen that the condition in (37) is non-linear in the design parameters A_{3j}, B_{3j}, K_j and P_{ji}. In no-delay systems, these types of non-linearities have been eliminated by some appropriate change of control variables with the general form of P_{ji} as follows (Gahinet 1996, Masubuchi *et al*. 1998):

$$P_{ji} = \begin{bmatrix} P_{1ji} & P_{2ji} \\ P_{2ji}^T & P_{3ji} \end{bmatrix}, \quad \forall j, i \in S. \tag{17}$$

When dealing with the output feedback control problem for time-delay systems, there are always some parameters coupled with their inverse which is required to be fixed *a priori*, see, e.g., Esfahani and Petersen (2000) and Xu and Chen (2004). In this paper, if we partition P_{ji} as (17) and use the linearizing change of variable approach as in Xu and Chen (2004) for condition (37), the design parameters $Y_{ji}, X_{1ji}^{1}, Y_{ji}^{-1}, X_{1ji}^{1^{-1}}$ will occur in the same inequality. Then, if we were to transfer the control design problem into the framework of LMI, we have to fix these parameters *a priori*, which makes the obtaining of the optimal relationships between the terms in the Leibniz-Newton formula (26), (27) almost impossible.

To obtain an easier design technique, we choose P_{ji} to be diagonal block matrices

$$P_{ji} = \begin{bmatrix} P_{1ji} & 0 \\ 0 & P_{2ji} \end{bmatrix}, \quad \forall j, i \in S.$$

Though, such a block-diagonal Lyapunov matrix is a special case, it is reasonable to choose Lyapunov parameters P_{1ji} for plant states $x(t)$ and P_{2ji} for control systems states $\hat{x}(t)$, respectively. Subsequently, we can obtain the optimal free weighting matrices by solving the corresponding linear matrix inequalities without the need to fix any design parameters, and the corresponding results were less conservative than the existing ones.

4. Robust H_∞ disturbance attenuation

In this section, we consider robust H_∞ disturbance attenuation for the time-delayed uncertain jump linear systems (11).

Theorem 2: *The time-delayed uncertain jump linear systems* (11) *is stochastically stable with* γ*-disturbance* H_∞ *attenuation* (12), *and the output feedback control law* (9) *is robust if there exist symmetric positive-definite matrices* P_{1ji}, P_{2ji}, Q, Z, *symmetric positive semi-definite matrices* $\bar{X}_{ji} \geq 0$, *constants* $\rho_{1ji} > 0$, $\rho_{2ji} > 0$, $\rho_{3ji} > 0$ *and appropriately dimensioned matrices* K_j, Y_{ji}, T_{ji}, N_{ji} *such that*

$$\begin{bmatrix}
\bar{L}_1 & \mu A_{1i}^T \hat{Z} & \hat{P}_{1ji} B_{1i} + \rho_{3ji} E_{1i}^T E_{3i} & 0 & C_i^T & \hat{P}_{1ji} H_{1i} & 0 & 0 \\
\bar{L}_2 & 0 & 0 & K_j^T & 0 & 0 & V_{ji}^T H_{2i} & K_j^T \\
\bar{L}_3 & \mu A_{2i}^T \hat{Z} & \rho_{3ji} E_{2i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\
\bar{L}_4 & \mu B_{2i}^T \hat{Z} & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{L}_5 & -\mu \hat{Z} & \mu \hat{Z} B_{1i} & 0 & 0 & \mu \hat{Z} H_{1i} & 0 & 0 \\
\bar{L}_6 & \mu B_{1i}^T \hat{Z} & -I + \rho_{3ji} E_{3i}^T E_{3i} & 0 & 0 & 0 & 0 & 0 \\
\bar{L}_7 & 0 & 0 & -I + \rho_{1ji} \alpha_i^2 I & 0 & 0 & 0 & 0 \\
\bar{L}_8 & 0 & 0 & 0 & -\rho_{4ji} I & 0 & H_{2i} & 0 \\
\bar{L}_9 & \mu H_{1i}^T \hat{Z} & 0 & 0 & 0 & -\rho_{3ji} I & 0 & 0 \\
\bar{L}_{10} & 0 & 0 & 0 & H_{2i}^T & 0 & -\rho_{2ji} I & 0 \\
\bar{L}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{1ji} I
\end{bmatrix} < 0 \tag{18}$$

$$\bar{\Gamma}_{ji} = \begin{bmatrix} \hat{X}_{11ji} & \hat{X}_{12ji} & \hat{X}_{13ji} & I_0^T \hat{Y}_{ji} \\ \hat{X}_{12ji}^T & \hat{X}_{22ji} & \hat{X}_{23ji} & \hat{T}_{ji} \\ \hat{X}_{13ji}^T & \hat{X}_{23ji}^T & \hat{X}_{33ji} & \hat{N}_{ji} \\ \hat{Y}_{ji}^T I_0 & \hat{T}_{ji}^T & \hat{N}_{ji}^T & \hat{Z} \end{bmatrix} \geq 0, \quad \forall i, j \in S, \tag{19}$$

where

$$\begin{bmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \\ \bar{L}_4 \\ \bar{L}_5 \\ \bar{L}_6 \\ \bar{L}_7 \\ \bar{L}_8 \\ \bar{L}_9 \\ \bar{L}_{10} \\ \bar{L}_{11} \end{bmatrix} = \begin{bmatrix} \Psi_{11} + \rho_{3ji}E_{1i}^T E_{1i} + \rho_{2ji}E_{4i}^T E_{4i} & C_i^T V_{ji} + \mu \hat{X}_{11ji}^2 & \Psi_{13} + \rho_{3ji}E_{1i}^T E_{2i} & \Psi_{14} \\ \mu \hat{X}_{11ji}^{2^T} + V_{ji}^T C_i & U_i + U_i^T + \Psi_{22} & \mu \hat{X}_{12ji}^2 & \mu \hat{X}_{13ji}^2 \\ \Psi_{13}^T + \rho_{3ji}E_{2i}^T E_{1i} & \mu \hat{X}_{12ji}^{2^T} & \Psi_{33} + \rho_{3ji}E_{2i}^T E_{2i} & \Psi_{34} \\ \Psi_{14}^T & \mu \hat{X}_{13ji}^{2^T} & \Psi_{34}^T & \Psi_{44} - \gamma^2 I \\ \mu \hat{Z} A_{1i} & 0 & \mu \hat{Z} A_{2i} & \mu \hat{Z} B_{2i} \\ B_{1i}^T \hat{P}_{1ji} + \rho_{3ji} E_{3i}^T E_{1i} & 0 & \rho_{3ji} E_{3i}^T E_{2i} & 0 \\ 0 & K_j & 0 & 0 \\ C_i & 0 & 0 & 0 \\ H_{1i}^T \hat{P}_{1ji} & 0 & 0 & 0 \\ 0 & H_{2i}^T V_{ji} & 0 & 0 \\ 0 & K_j & 0 & 0 \end{bmatrix}$$

$$\bar{X}_{ji} = \begin{bmatrix} \bar{X}_{11ji} & \bar{X}_{12ji} & \bar{X}_{13ji} \\ \bar{X}_{12ji}^T & \bar{X}_{22ji} & \bar{X}_{23ji} \\ \bar{X}_{13ji}^T & \bar{X}_{23ji}^T & \bar{X}_{33ji} \end{bmatrix} = \begin{bmatrix} \bar{X}_{11ji}^1 & \bar{X}_{11ji}^2 & \bar{X}_{12ji}^1 & \bar{X}_{13ji}^1 \\ \bar{X}_{11ji}^{2^T} & \bar{X}_{11ji}^3 & \bar{X}_{12ji}^2 & \bar{X}_{13ji}^2 \\ \bar{X}_{12ji}^{1^T} & \bar{X}_{12ji}^{2^T} & \bar{X}_{22ji} & \bar{X}_{23ji} \\ \bar{X}_{13ji}^{1^T} & \bar{X}_{13ji}^{2^T} & \bar{X}_{23ji}^T & \bar{X}_{33ji} \end{bmatrix}$$

with

$$\Psi_{11} = \begin{cases} A_{1i}^T \hat{P}_{1ii} + \hat{P}_{1ii} A_{1i} + \hat{Y}_{ii} + \hat{Y}_{ii}^T + (1+\eta\mu)\hat{Q} + \mu \hat{X}_{11ii}^1 + \sum_{j=1}^N q_{ij} \hat{P}_{1ij} + \sum_{j=1}^N q_{ij}^1 \hat{P}_{1ji}, & j = i \\ A_{1i}^T \hat{P}_{1ji} + \hat{P}_{1ji} A_{1i} + \hat{Y}_{ji} + \hat{Y}_{ji}^T + (1+\eta\mu)\hat{Q} + \mu \hat{X}_{11ji}^1 + q_{ji}^0 (\hat{P}_{1ii} - \hat{P}_{1ji}), & j \neq i \end{cases}$$

$$\Psi_{22} = \begin{cases} \sum_{j=1}^N q_{ij} \hat{P}_{2ij} + \sum_{j=1}^N q_{ij}^1 \hat{P}_{2ji} + \mu \hat{X}_{11ii}^3, & \text{if } j = i \\ q_{ji}^0 (\hat{P}_{2ii} - \hat{P}_{2ji}) + \mu \hat{X}_{11ji}^3, & \text{if } j \neq i \end{cases}$$

$$\Psi_{13} = \hat{P}_{1ji} A_{2i} - \hat{Y}_{ji} + \hat{T}_{ji}^T + \mu \hat{X}_{12ji}^1, \quad \Psi_{14} = \hat{P}_{1ji} B_{2i} + \hat{N}_{ji}^T + \mu \hat{X}_{13ji}^1,$$

$$\Psi_{33} = -\hat{T}_{ji} - \hat{T}_{ji}^T - (1-h_i)\hat{Q} + \mu \hat{X}_{22ji}, \quad \Psi_{34} = -\hat{N}_{ji}^T + \mu \hat{X}_{23ji}, \quad \Psi_{41} = \mu \hat{X}_{33ji},$$

$$\eta = \max_{i \in S} \{q_{ii}|\}, \quad V_{ji} = B_{3j}^T \hat{P}_{2ji}, \quad U_{ji} = A_{3j}^T \hat{P}_{2ji}.$$

$$\begin{bmatrix} \hat{P}_{1ji} & \hat{P}_{2ji} & \hat{Q} & \hat{Z} & \hat{Y}_{ji} & \hat{T}_{ji} & \hat{N}_{ji} \end{bmatrix} = \rho_{4ji}^{-1} \begin{bmatrix} P_{1ji} & P_{2ji} & Q & Z & Y_{ji} & T_{ji} & N_{ji} \end{bmatrix},$$

$$\begin{bmatrix} \hat{X}_{11ji}^1 & \hat{X}_{11ji}^2 & \hat{X}_{12ji}^1 & \hat{X}_{13ji}^1 \\ \hat{X}_{11ji}^{2^T} & \hat{X}_{11ji}^3 & \hat{X}_{12ji}^2 & \hat{X}_{13ji}^2 \\ \hat{X}_{12ji}^{1^T} & \hat{X}_{12ji}^{2^T} & \hat{X}_{22ji} & \hat{X}_{23ji} \\ \hat{X}_{13ji}^{1^T} & \hat{X}_{13ji}^{2^T} & \hat{X}_{23ji}^T & \hat{X}_{33ji} \end{bmatrix} = \rho_{4ji}^{-1} \begin{bmatrix} \bar{X}_{11ji}^1 & \bar{X}_{11ji}^2 & \bar{X}_{12ji}^1 & \bar{X}_{13ji}^1 \\ \bar{X}_{11ji}^{2^T} & \bar{X}_{11ji}^3 & \bar{X}_{12ji}^2 & \bar{X}_{13ji}^2 \\ \bar{X}_{12ji}^{1^T} & \bar{X}_{12ji}^{2^T} & \bar{X}_{22ji} & \bar{X}_{23ji} \\ \bar{X}_{13ji}^{1^T} & \bar{X}_{13ji}^{2^T} & \bar{X}_{23ji}^T & \bar{X}_{33ji} \end{bmatrix}.$$

The proof of this theorem is put into Appendix B.

In the case when the jumping parameter process can be directly and precisely measured; that is, $p_{mj,t} = p^o_{mj,t}$, $\forall t \in [0, \infty)$, and the closed-loop system (11) is specialized as

$$\begin{cases} \dot{\xi}(t) = \tilde{A}_1(p_{mj,t}, t)\xi(t) + \tilde{A}_2(p_{mj,t})I_0\xi(t - \tau_{p_{mj,t}}(t)) \\ \qquad + \tilde{B}_2(p_{mj,t})w(t) \\ z(t) = \left[C(p_{mj,t}) + \Delta_C(p_{mj,t}, t)\right]I_0\xi(t) \\ I_0\xi(s) = f(s), \quad p_{mj,s} = p_{mj,0}, \quad s \in [-2\mu, 0], \end{cases} \tag{20}$$

where

$$\tilde{A}_{1ji} = \begin{bmatrix} A_{1i} + \Delta_{A_{1i}}(t) & (B_{1i} + \Delta_{B_{1i}}(t))(I + \alpha_i\phi_i(t))K_i \\ B_{3i}(C_i + \Delta_{C_i}(t)) & A_{3i} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$$

$$\tilde{A}_{2i} = \begin{bmatrix} A_{2i} + \Delta_{A_{2i}}(t) \\ 0 \end{bmatrix} \in \mathbb{R}^{2n \times n}, \quad \tilde{B}_{2i} = \begin{bmatrix} B_{2i} \\ 0 \end{bmatrix} \in \mathbb{R}^{2n \times m_2},$$

$$I_0 = \begin{bmatrix} I & 0 \end{bmatrix} \in \mathbb{R}^{n \times 2n}$$

for each $p_{mj,t} = i$, $\forall i \in S$, then by Theorem 2, we have the following corollary.

Corollary 1: *The time-delayed uncertain jump linear systems* (11) *is stochastically stable with γ-disturbance H_∞ attenuation* (12), *and the output feedback control law* (9) *is robust if the jumping parameter process can be directly and precisely measured, and there exist symmetric positive-definite matrices P_{1i}, P_{2i}, Q, Z, symmetric positive semi-definite matrices $\tilde{X}_i \geq 0$, constant $\rho_{1i} > 0$, $\rho_{2i} > 0$, $\rho_{3i} > 0$ and appropriately dimensioned matrices K_i, Y_i, T_i, N_i such that*

$$\begin{bmatrix} \tilde{L}_1 & \mu A_{1i}^T\hat{Z} & \hat{P}_{1i}B_{1i} + \rho_{3i}E_{1i}^TE_{3i} & 0 & C_i^T & \hat{P}_{1i}H_{1i} & 0 & 0 \\ \tilde{L}_2 & 0 & 0 & K_i^T & 0 & 0 & V_i^TH_{2i} & K_i^T \\ \tilde{L}_3 & \mu A_{2i}^T\hat{Z} & \rho_{3i}E_{2i}^TE_{3i} & 0 & 0 & 0 & 0 & 0 \\ \tilde{L}_4 & \mu B_{2i}^T\hat{Z} & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{L}_5 & -\mu\hat{Z} & \mu\hat{Z}B_{1i} & 0 & 0 & \mu\hat{Z}H_{1i} & 0 & 0 \\ \tilde{L}_6 & \mu B_{1i}^T\hat{Z} & -I + \rho_{3i}E_{3i}^TE_{3i} & 0 & 0 & 0 & 0 & 0 \\ \tilde{L}_7 & 0 & 0 & -I + \rho_{1i}\alpha_i^2I & 0 & 0 & 0 & 0 \\ \tilde{L}_8 & 0 & 0 & 0 & -\rho_{4i}I & 0 & H_{2i} & 0 \\ \tilde{L}_9 & \mu H_{1i}^T\hat{Z} & 0 & 0 & 0 & -\rho_{3i}I & 0 & 0 \\ \tilde{L}_{10} & 0 & 0 & 0 & H_{2i}^T & 0 & -\rho_{2i}I & 0 \\ \tilde{L}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{1i}I \end{bmatrix} < 0 \tag{21}$$

$$\tilde{\Gamma}_i = \begin{bmatrix} \hat{X}_{11i} & \hat{X}_{12i} & \hat{X}_{13i} & I_0^T\hat{Y}_i \\ \hat{X}_{12i}^T & \hat{X}_{22i} & \hat{X}_{23i} & \hat{T}_i \\ \hat{X}_{13i}^T & \hat{X}_{23i}^T & \hat{X}_{33i} & \hat{N}_i \\ \hat{Y}_i^TI_0 & \hat{T}_i^T & \hat{N}_i^T & \hat{Z} \end{bmatrix} \geq 0, \quad \forall i \in S, \tag{22}$$

where

$$\begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \tilde{L}_3 \\ \tilde{L}_4 \\ \tilde{L}_5 \\ \tilde{L}_6 \\ \tilde{L}_7 \\ \tilde{L}_8 \\ \tilde{L}_9 \\ \tilde{L}_{10} \\ \tilde{L}_{11} \end{bmatrix} = \begin{bmatrix} \tilde{\Psi}_{11} + \rho_{3i}E_{1i}^TE_{1i} + \rho_{2i}E_{4\iota}^TE_{4i} & C_i^TV_i + \mu\hat{X}_{11i}^2 & \tilde{\Psi}_{13} + \rho_{3i}E_{1i}^TE_{2\iota} & \tilde{\Psi}_{14} \\ \mu\hat{X}_{11i}^{2^T} + V_i^TC_i & U_i + U_i^T + \tilde{\Psi}_{22} & \mu\hat{X}_{12i}^2 & \mu\hat{X}_{13i}^2 \\ \tilde{\Psi}_{13}^T + \rho_{3i}E_{2i}^TE_{1i} & \mu\hat{X}_{12i}^{2^T} & \tilde{\Psi}_{33} + \rho_{3i}E_{2i}^TE_{2i} & \tilde{\Psi}_{34} \\ \tilde{\Psi}_{14}^T & \mu\hat{X}_{13i}^{2^T} & \tilde{\Psi}_{34}^T & \tilde{\Psi}_{44} - \gamma^2I \\ \mu\hat{Z}A_{1i} & 0 & \mu\hat{Z}A_{2i} & \mu\hat{Z}B_{2i} \\ B_{1i}^TP_{2i} + \rho_{33i}E_{3i}^TE_{1i} & 0 & \rho_{3i}E_{3i}^TE_{2i} & 0 \\ 0 & K_i & 0 & 0 \\ C_i & 0 & 0 & 0 \\ H_{1i}^T\hat{P}_{1i} & 0 & 0 & 0 \\ 0 & H_{2i}^TV_i & 0 & 0 \\ 0 & K_i & 0 & 0 \end{bmatrix}$$

$$\tilde{X}_i = \begin{bmatrix} \tilde{X}_{11i} & \tilde{X}_{12i} & \tilde{X}_{13i} \\ \tilde{X}_{12i}^T & \tilde{X}_{22i} & \tilde{X}_{23i} \\ \tilde{X}_{13i}^T & \tilde{X}_{23i}^T & \tilde{X}_{33i} \end{bmatrix} = \begin{bmatrix} \tilde{X}_{11i}^1 & \tilde{X}_{11i}^2 & \tilde{X}_{12i}^1 & \tilde{X}_{13i}^1 \\ \tilde{X}_{11i}^{2^T} & \tilde{X}_{11i}^3 & \tilde{X}_{12i}^2 & \tilde{X}_{13i}^2 \\ \tilde{X}_{12i}^{1^T} & \tilde{X}_{12i}^{2^T} & \tilde{X}_{22i} & \tilde{X}_{23i} \\ \tilde{X}_{13i}^{1^T} & \tilde{X}_{13i}^{2^T} & \tilde{X}_{23i}^T & \tilde{X}_{33i} \end{bmatrix}$$

with

$$\tilde{\Psi}_{11} = A_{1i}^T\hat{P}_{1i} + \hat{P}_{1i}A_{1i} + \hat{Y}_i + \hat{Y}_i^T + (1+\eta\mu)\hat{Q} + \mu\hat{X}_{11i}^1 + \sum_{j=1}^{N} q_{ij}\hat{P}_{1j}$$

$$\tilde{\Psi}_{22} = \sum_{j=1}^{N} q_{ij}\hat{P}_{2j} + \mu\hat{X}_{11i}^2, \quad \tilde{\Psi}_{13} = \hat{P}_{1i}A_{2i} - \hat{Y}_i + \hat{T}_i^T + \mu\hat{X}_{12i}^1,$$

$$\tilde{\Psi}_{14} = \hat{P}_{1i}B_{2i} + \hat{N}_i^T + \mu\hat{X}_{13i}^1, \quad \tilde{\Psi}_{33} = -\hat{T}_i - \hat{T}_i^T - (1-h_i)\hat{Q} + \mu\hat{X}_{22i},$$

$$\tilde{\Psi}_{34} = -\hat{N}_i^T + \mu\hat{X}_{23i} \quad \tilde{\Psi}_{44} = \mu\hat{X}_{33i}, \quad \eta = \max_{i\in S}\{q_{ii}|q_{ii}|\}, \quad V_i = B_{3i}^T\hat{P}_{2i}, \quad U_i = A_{3i}^T\hat{P}_{2i},$$

$$\left[\hat{P}_{1i} \;\; \hat{P}_{2i} \;\; \hat{Q} \;\; \hat{Z} \;\; \hat{Y}_i \;\; \hat{T}_i \;\; \hat{N}_i\right] = \rho_{4i}^{-1}\left[P_{1i} \;\; P_{2i} \;\; Q \;\; Z \;\; Y_i \;\; T_i \;\; N_i\right],$$

$$\begin{bmatrix} \hat{X}_{11i}^1 & \hat{X}_{11i}^2 & \hat{X}_{12i}^1 & \hat{X}_{13i}^1 \\ \hat{X}_{11i}^{2^T} & \hat{X}_{11i}^3 & \hat{X}_{12i}^2 & \hat{X}_{13i}^2 \\ \hat{X}_{12i}^{1^T} & \hat{X}_{12i}^{2^T} & \hat{X}_{22i} & \hat{X}_{23i} \\ \hat{X}_{13i}^{1^T} & \hat{X}_{13i}^{2^T} & \hat{X}_{23i}^T & \hat{X}_{33i} \end{bmatrix} = \rho_{4i}^{-1}\begin{bmatrix} \tilde{X}_{11i}^1 & \tilde{X}_{11i}^2 & \tilde{X}_{12i}^1 & \tilde{X}_{13i}^1 \\ \tilde{X}_{11i}^{2^T} & \tilde{X}_{11i}^3 & \tilde{X}_{12i}^2 & \tilde{X}_{13i}^2 \\ \tilde{X}_{12i}^{1^T} & \tilde{X}_{12i}^{2^T} & \tilde{X}_{22i} & \tilde{X}_{23i} \\ \tilde{X}_{13i}^{1^T} & \tilde{X}_{13i}^{2^T} & \tilde{X}_{23i}^T & \tilde{X}_{33i} \end{bmatrix}.$$

5. Simulation

To illustrate the usefulness of the above theory, let us consider the following examples.

Example 1: Consider a time-delayed uncertain jump linear system (11) in $\mathbb{R}^2$ with two regimes $p_{mj,t} \in S = \{1,2\}$. For Mode 1, the dynamics of the system are described as

$$A_{11} = \begin{bmatrix} -7 & -5 \\ 0 & -8 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -4 & 3 \\ -1 & 1 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 3 \\ 1 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}^T,$$

$$H_{11} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}^T,$$

$$E_{21} = \begin{bmatrix} 0.1 \\ 2 \end{bmatrix}^T, \quad E_{31} = -0.5, \quad H_{21} = 1.$$

$$E_{11} = \begin{bmatrix} 0.9 \\ 0.2 \end{bmatrix}^T, \quad \mu_1 = 0.1, \quad h_1 = 0.5, \quad \alpha_1 = 2.$$

For Mode 2, the dynamics of the system are described as

$$A_{12} = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -4 & 0 \\ 1 & -5 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}^T,$$

$$H_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}^T, \quad E_{22} = \begin{bmatrix} -1 \\ 0.2 \end{bmatrix}^T,$$

$$E_{32} = 0.6, \quad H_{22} = 2$$

$$E_{42} = \begin{bmatrix} -1 \\ 0.2 \end{bmatrix}^T, \quad \mu_2 = 0.1, \quad h_2 = 0.4, \quad \alpha_2 = 3.$$

We assume that the noise attenuation level $\gamma = 1.2$, and suppose that the matrices of transition rates are given by

$$[q_{ij}]_{2\times 2} = \begin{bmatrix} -10 & 10 \\ 20 & -20 \end{bmatrix}, \quad [q_{ij}^0]_{2\times 2} = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix},$$

$$[q_{ij}^1]_{2\times 2} = \begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix}.$$

Solving the LMIs in (18) and (19), we obtain the solution as follows:

$$\hat{P}_{111}=\begin{bmatrix}0.756 & -0.175\\ -0.175 & 0.443\end{bmatrix},\quad \hat{P}_{112}=\begin{bmatrix}0.148 & -0.067\\ -0.067 & 0.274\end{bmatrix},$$

$$\hat{P}_{121}=\begin{bmatrix}1.000 & -0.117\\ -0.117 & 0.510\end{bmatrix},\quad \hat{P}_{122}=\begin{bmatrix}2.895 & -1.453\\ -1.453 & 2.255\end{bmatrix},$$

$$\hat{T}_{11}=\begin{bmatrix}7.138 & -5.667\\ -5.705 & 9.594\end{bmatrix},\quad \hat{T}_{12}=\begin{bmatrix}6.153 & -4.726\\ -5.380 & 9.944\end{bmatrix},$$

$$\hat{P}_{211}=\begin{bmatrix}4.337 & 0.807\\ 0.807 & 2.865\end{bmatrix},\quad \hat{P}_{212}=\begin{bmatrix}2.573 & 0.562\\ 0.562 & 2.041\end{bmatrix},$$

$$\hat{P}_{221}=\begin{bmatrix}3.860 & 2.116\\ 2.116 & 3.539\end{bmatrix},\quad \hat{P}_{222}=\begin{bmatrix}8.647 & 5.647\\ 5.647 & 7.599\end{bmatrix},$$

$$\hat{T}_{21}=\begin{bmatrix}6.446 & -4.977\\ -4.999 & 8.870\end{bmatrix},\quad \hat{T}_{22}=\begin{bmatrix}6.295 & -5.008\\ -5.105 & 9.406\end{bmatrix},$$

$$\hat{Y}_{11}=\begin{bmatrix}-8.435 & 6.694\\ 5.939 & -9.778\end{bmatrix},\quad \hat{Y}_{12}=\begin{bmatrix}-6.146 & 4.713\\ 5.388 & -9.958\end{bmatrix},$$

$$\hat{Y}_{21}=\begin{bmatrix}-7.943 & 6.128\\ 4.893 & -8.789\end{bmatrix},\quad \hat{Y}_{22}=\begin{bmatrix}-5.568 & 3.665\\ 5.459 & -10.057\end{bmatrix},$$

$$\hat{Q}=\begin{bmatrix}0.003 & 0.001\\ 0.001 & 0.001\end{bmatrix},\quad \hat{Z}=\begin{bmatrix}0.601 & -0.474\\ -0.474 & 0.872\end{bmatrix},$$

$$U_{11}=\begin{bmatrix}-76.958 & -0.589\\ -0.583 & -80.969\end{bmatrix},$$

$$U_{12}=\begin{bmatrix}-173.858 & -1.063\\ -1.187 & -157.808\end{bmatrix},$$

$$K_1=\begin{bmatrix}1.624\\ -1.380\end{bmatrix}^T,\quad U_{21}=\begin{bmatrix}-42.435 & -3.346\\ 2.060 & -45.679\end{bmatrix},$$

$$U_{22}=\begin{bmatrix}-106.429 & 8.381\\ 8.269 & -115.244\end{bmatrix},\quad K_2=\begin{bmatrix}-0.843\\ 2.180\end{bmatrix}^T,$$

$$V_{11}=\begin{bmatrix}-0.577\\ -1.018\end{bmatrix}^T,\quad V_{12}=\begin{bmatrix}-0.621\\ -0.165\end{bmatrix}^T,$$

$$V_{21}=\begin{bmatrix}-0.631\\ -0.584\end{bmatrix}^T,\quad V_{22}=\begin{bmatrix}-4.111\\ -3.695\end{bmatrix}^T,$$

$$\hat{N}_{11}=\begin{bmatrix}0.899\\ -0.712\end{bmatrix}^T,\quad \hat{N}_{12}=\begin{bmatrix}-0.793\\ 1.464\end{bmatrix}^T,$$

$$\hat{N}_{21}=\begin{bmatrix}1.327\\ -1.020\end{bmatrix}^T,\quad \hat{N}_{22}=\begin{bmatrix}-0.624\\ 1.149\end{bmatrix}^T,$$

$\rho_{111}=0.167$, $\rho_{112}=0.078$, $\rho_{121}=0.167$,
$\rho_{122}=0.079$, $\rho_{211}=0.609$, $\rho_{311}=0.276$,
$\rho_{212}=0.153$, $\rho_{221}=0.252$,, $\rho_{222}=5.585$,
$\rho_{312}=0.073$, $\rho_{321}=0.348$, $\rho_{322}=0.582$,
$\rho_{411}=9.452$, $\rho_{412}=48.758$, $\rho_{421}=19.488$,
$\rho_{422}=4.182$,

$$\hat{X}_{11}=\begin{bmatrix}152.418 & -90.472 & 4.756 & 8.305 & -116.994 & 87.333 & -20.875\\ -90.472 & 178.250 & -9.403 & -16.477 & 92.267 & -160.958 & 9.657\\ 4.756 & -9.403 & 996.581 & 107.227 & -0.030 & 0.023 & 0.025\\ 8.305 & -16.477 & 107.227 & 983.803 & -0.033 & 0.018 & 0.027\\ -116.994 & 92.267 & -0.030 & -0.033 & 111.911 & -91.689 & 9.115\\ 87.333 & -160.958 & 0.023 & 0.018 & -91.689 & 150.255 & -7.557\\ -20.875 & 9.657 & 0.025 & 0.027 & 9.115 & -7.557 & 8.156\end{bmatrix}$$

$$\hat{X}_{12}=\begin{bmatrix}91.843 & -70.944 & 17.690 & 4.776 & -91.921 & 76.319 & 7.531\\ -70.944 & 129.860 & 8.860 & 2.386 & 71.407 & -133.050 & -21.410\\ 17.690 & 8.860 & 1712.300 & 144.210 & 0.001 & -0.010 & -0.004\\ 4.776 & 2.386 & 144.210 & 1736.900 & -0.004 & -0.001 & 0.002\\ -91.921 & 71.407 & 0.001 & -0.004 & 92.462 & -76.621 & -7.613\\ 76.319 & -133.050 & -0.010 & -0.001 & -76.621 & 137.410 & 21.320\\ 7.531 & -21.410 & -0.004 & 0.002 & -7.613 & 21.320 & 8.804\end{bmatrix}$$

$$\hat{X}_{21} = \begin{bmatrix} 119.168 & -68.442 & 5.190 & 4.829 & -96.480 & 71.394 & -24.073 \\ -68.442 & 170.693 & -10.160 & -9.490 & 80.542 & -149.340 & 9.663 \\ 5.190 & -10.160 & 494.440 & 150.705 & 0.004 & -0.293 & -0.007 \\ 4.829 & -9.490 & 150.705 & 334.947 & -0.015 & -0.201 & -0.006 \\ -96.480 & 80.542 & 0.004 & -0.015 & 100.234 & -80.560 & 13.351 \\ 71.394 & -149.340 & -0.293 & -0.201 & -80.559 & 136.532 & -10.419 \\ -24.073 & 9.663 & -0.007 & -0.006 & 13.351 & -10.419 & 9.497 \end{bmatrix}$$

$$\hat{X}_{22} = \begin{bmatrix} 251.674 & -176.139 & 66.028 & 59.345 & -4.696 & 20.864 & -5.000 \\ -176.139 & 259.635 & 32.979 & 29.641 & 7.326 & -70.281 & -18.647 \\ 66.028 & 32.979 & 2044.842 & 605.962 & -0.001 & -0.010 & 0.004 \\ 59.345 & 29.641 & 605.962 & 1634.627 & -0.001 & -0.008 & 0.004 \\ -4.696 & 7.326 & -0.001 & -0.001 & 90.505 & -73.603 & -6.343 \\ 20.864 & -70.281 & -0.010 & -0.008 & -73.603 & 141.486 & 13.189 \\ -5.000 & -18.647 & 0.004 & 0.004 & -6.343 & 13.189 & 9.660 \end{bmatrix}.$$

Therefore, by Theorem 2, the corresponding parameters of a suitable robust output feedback control law (9) can be chosen as

$$A_{311} = \begin{bmatrix} -18.683 & 5.405 \\ 5.055 & -29.781 \end{bmatrix},$$

$$A_{312} = \begin{bmatrix} -71.775 & 17.470 \\ 19.233 & -82.112 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 1.624 \\ -1.380 \end{bmatrix}^T,$$

$$A_{321} = \begin{bmatrix} -15.584 & 11.321 \\ 8.373 & -19.677 \end{bmatrix},$$

$$A_{322} = \begin{bmatrix} -25.311 & 21.099 \\ 19.912 & -30.844 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.843 \\ 2.180 \end{bmatrix}^T,$$

$$B_{311} = \begin{bmatrix} -0.071 \\ -0.336 \end{bmatrix}, \quad B_{312} = \begin{bmatrix} -0.238 \\ -0.015 \end{bmatrix},$$

$$B_{321} = \begin{bmatrix} -0.109 \\ -0.100 \end{bmatrix}, \quad B_{322} = \begin{bmatrix} -0.307 \\ -0.258 \end{bmatrix}.$$

Example 2: Consider the robust stability of the uncertain system (1) with the following parameters:

$$A_{11} = \begin{bmatrix} a_{11} & 0 \\ 1.4 & -10 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -3 & -0.8 \\ 0.2 & -2 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 1 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$B_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0.1 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad H_{12} = \begin{bmatrix} 0 & 1 \\ 0 & -0.3 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E_{12} = \begin{bmatrix} -1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0 & 0.3 \\ 0 & -1 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} -0.2 & 0 \\ 1 & 0 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad H_{21} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad H_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$H_{31} = \begin{bmatrix} 0 & 0.2 \\ 0 & 1 \end{bmatrix}, \quad H_{32} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$[q_{ij}]_{2\times 2} = \begin{bmatrix} -2 & 2 \\ 5 & -5 \end{bmatrix}.$$

This example was given in Boukas and Liu (2002) for "$a_{11} = -1$". To compare with Theorem 9.18 in Boukas and Liu (2002), Theorem 1 should be reduced to the conditions that the jumping parameter process can be directly and precisely measured and controller can be accurately implemented. Furthermore, we also assumed that

$$h_1 = h_2 = h$$
$$\Delta_{A_2}(p_{mj,t}, t) = H_3(p_{mj,t})F(p_{mj,t}, t)E_2(p_{mj,t})$$
$$\Delta_C(p_{mj,t}, t) = H_2(p_{mj,t})F(p_{mj,t}, t)E_1(p_{mj,t}).$$

The corresponding results are similar to Corollary 1, and are omitted here. The maximum allowed value of time delay for different "h" obtained from Theorem 1

Table 1. The maximum allowed value of time delay (μ).

h		0	0.2	0.5	0.9
$a_{11}=-1$	E. K. Boukas (2002)	0.1830	0.1064	–	–
	Theorem 3.1	0.7012	0.6916	0.6732	0.6326
$a_{11}=-10$	E. K. Boukas (2002)	1.5745	1.3378	0.7780	–
	Theorem 3.1	1.8474	1.5993	0.9258	0.4949

are shown in table 1, for comparison. The table also lists the results obtained from Theorem 9.18 in Boukas and Liu (2002). From the example, we can find that our results show much less conservatism than those in Boukas and Liu (2002) especially for the increasing of the value of h.

6. Conclusion

The problem of robust output feedback H_∞ control for time-delayed uncertain jump linear systems has been studied. We have presented sufficient conditions on the existence of output feedback control directly by the imperfect information $p^o_{mj,t}$, which guarantees not only the robust exponential mean-square stability but also the γ-disturbance H_∞ attenuation for the closed loop system for all admissible parameter uncertainties and time delays. However, all of these results are established under condition of a prior knowledge of the upper bounds of the system uncertainties. A possible direction for future work is to obtain adaptive H_∞ control laws with less knowledge of those bounds.

Appendix A. Proof of Theorem 1

Proof: Consider the nominal time-delayed jump linear system Σ_0 without disturbance

$$\Sigma_0:\begin{cases}\dot{\xi}(t)=\hat{A}_1(p^o_{mj,t},p_{mj,t})\xi(t)+\hat{A}_2(p_{mj,t})I_0\xi(t-\tau_{p_{mj,t}}(t))\\ z(t)=\left[C(p_{mj,t})+\Delta_C(p_{mj,t},t)\right]I_0\xi(t)\\ I_0\xi(s)=f(s),\quad p_{mj,s}=p_{mj,0},\quad s\in[-2\mu,0],\end{cases}\tag{23}$$

where

$$\hat{A}_1(p^o_{mj,t},p_{mj,t})=\begin{bmatrix}A_1(p_{mj,t}) & B_1(p_{mj,t})K(p^o_{mj,t})\\ B_3(p^o_{mj,t})C(p_{mj,t}) & A_3(p^o_{mj,t})\end{bmatrix}\in\mathbb{R}^{2n\times 2n},$$

$$\hat{A}_2(p_{mj,t})=\begin{bmatrix}A_2(p_{mj,t})\\ 0\end{bmatrix}\in\mathbb{R}^{2n\times n}.$$

Note that $\{(\xi(t),p^o_{mj,t},p_{mj,t}),t\geq 0\}$ is non-Markovian because of the existence of $\tau_{p_{mj,t}}(t)$. To transform (23) into the framework of Markov systems, define a process $\{(\xi_t,p^o_{mj,t},p_{mj,t}),t\geq 0)\}$ taking values in C_0 as follows:

$$\xi_t=\{\xi(\theta+t)|-2\mu\leq\theta\leq 0\},$$

where $C_0=\bigcup_{i,j\in S}\mathcal{C}[-2\mu,0]\times\{i,j\}$, and $C[-2\mu,0]$ represents the space of continuous functions on interval $[-2\mu,0]$. Then, similar to Xu *et al.* (2003), we can verify that $\{(\xi_t,p^o_{mj,t},p_{mj,t}),t\geq 0)\}$ is a strong Markov process with state space C_0.

Consider the following Lyapunov function candidate:

$$V(\xi_t,p^o_{mj,t},p_{mj,t},t)=V_1+V_2+V_3+V_4,\tag{24}$$

where

$$\begin{aligned}V_1&=\xi^T(t)P(p^o_{mj,t},p_{mj,t})\xi(t)\\&=x^T(t)P_1(p^o_{mj,t},p_{mj,t})x(t)+\hat{x}^T(t)P_2(p^o_{mj,t},p_{mj,t})\hat{x}(t)\\ V_2&=\int_{t-\tau_{p_{mj,t}}(t)}^t x^T(s)Qx(s)\,\mathrm{d}s=\int_{t-\tau_{p_{mj,t}}}^t\xi^T(s)I_0^TQI_0\xi(s)\,\mathrm{d}s\\ V_3&=\eta\int_{-\mu}^0\int_{t-\theta}^t x^T(s)Qx(s)\,\mathrm{d}s\,\mathrm{d}\theta\\&=\eta\int_{-\mu}^0\int_{t+\theta}^t\xi^T(s)I_0^TQI_0\xi(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ V_4&=\int_{-\mu}^0\int_{t+\theta}^t\dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\&=\int_{-\mu}^0\int_{t+\theta}^t\dot{\xi}^T(s)I_0^TQI_0\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta.\end{aligned}$$

From the definition of infinitesimal generator, $\mathscr{L}$, in Lemma 3, we have the following observations.

Case 1: $p^o_{mj,t}=p_{mj,t}=i$. It can be verified that

$$\begin{aligned}\mathscr{L}V_1&=\xi^T(t)\left[\hat{A}_{1ii}^TP_{ii}+P_{ii}\hat{A}_{1ii}+\sum_{j=1}^N q_{ij}P_{ij}+\sum_{j=1}^N q^1_{ij}P_{ji}\right]\xi(t)\\&\quad+\xi^T(t)P_{ii}\hat{A}_{2i}x(t-\tau_i(t))+x^T(t-\tau_i(t))\hat{A}_{2i}^TP_{ii}\xi(t)\\ \mathscr{L}V_2&=\xi^T(t)I_0^TQI_0\xi(t)-(1-\dot{\tau}_i(t))x^T(t-\tau_i(t))Qx(t-\tau_i(t))\\&\quad+\sum_{j=1}^N q_{ij}\int_{t-\tau_j(t)}^t x^T(s)Qx(s)\,\mathrm{d}s\end{aligned}$$

$$\leq \xi^T(t)I_0^TQI_0\xi(t) - (1-h_i)x^T(t-\tau_i(t))Qx(t-\tau_i(t)) + \sum_{j=1}^N q_{ij}\int_{t-\tau_j(t)}^t x^T(s)Qx(s)\,\mathrm{d}s$$

$$\mathscr{L}V_3 = \eta\mu\xi^T(t)I_0^TQI_0\xi(t) - \eta\int_{t-\mu}^t x^T(s)Qx(s)\,\mathrm{d}s$$

$$\mathscr{L}V_4 = \mu\dot{\xi}^T(t)I_0^TZI_0\dot{\xi}(t) - \int_{t-\mu}^t \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s \leq \mu\dot{\xi}^T(t)I_0^TZI_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^t \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s.$$

Noting (2) and (3), we have

$$\begin{aligned}\sum_{j=1}^N q_{ij}\int_{t-\tau_j(t)}^t x^T(s)Qx(s)\mathrm{d}s &\leq \sum_{j=1,j\neq i}^N q_{ij}\int_{t-\mu}^t x^T(s)Qx(s)\,\mathrm{d}s\\ &= -q_{ii}\int_{t-\mu}^t x^T(s)Qx(s)\,\mathrm{d}s\\ &\leq \eta\int_{t-\mu}^t x^T(s)Qx(s)\,\mathrm{d}s. \qquad (25)\end{aligned}$$

To overcome the conservativeness in selecting the optimal weighting matrices between the terms in the Leibniz–Newton formula, the following condition is presented (Wu *et al.* 2004):

$$2\left[x^T(t)Y + x^T(t-d(t))T\right] \times \left[x(t) - \int_{t-d(t)}^t \dot{x}(s)\,\mathrm{d}s - x(t-d(t))\right] = 0,$$

where the free weighting matrices Y and T indicate the relationship between the terms in the above formula, and they can easily be selected by means of linear matrix inequalities.

The following conditions are also employed to complete the proof:

$$\mu\zeta^T(t)X(p_{m,t}^o, p_{mj,t})\zeta(t) - \int_{t-\tau_{p_{mj,t}}(t)}^t \zeta^T(t)X(p_{mj,t}^o, p_{mj,t})\zeta(t)\,\mathrm{d}s \geq 0 \qquad (26)$$

$$2\left[\xi^T(t)I_0^TY(p_{mj,t}^o, p_{mj,t}) + x^T(t-\tau_{p_{mj,t}}(t))T(p_{mj,t}^o, p_{mj,t})\right] \times \left[I_0\xi(t) - \int_{t-\tau_{p_{mj,t}}(t)}^t \dot{x}(s)\mathrm{d}s - x(t-\tau_{p_{mj,t}}(t))\right] = 0, \qquad (27)$$

where $\zeta^T(t) = [\xi^T(t)\ x^T(t-\tau_{p_{mj,t}}(t))]$, and $X(p_{mj,t}^o, p_{mj,t})$ are defined in Theorem 1.

We have

$$\begin{aligned}&\mathscr{L}V(\xi_t, i, i, t)\\ &\leq \xi^T(t)\left[\hat{A}_{1ii}^TP_{ii} + P_{ii}\hat{A}_{1ii} + \sum_{j=1}^N q_{ij}P_{ij} + \sum_{j=1}^N q_{ij}^1P_{ji}\right]\xi(t)\\ &\quad + \xi^T(t)P_{ii}\hat{A}_{2i}x(t-\tau_i(t)) + x^T(t-\tau_i(t))\hat{A}_{2i}^TP_{ii}\xi(t)\\ &\quad + (1+\eta\mu)\xi^T(t)I_0^TQI_0\xi(t) - (1-h_i)x^T\\ &\quad \times(t-\tau_i(t))Qx(t-\tau_i(t))\\ &\quad + \mu\dot{\xi}^T(t)I_0^TZI_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^t \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s\\ &\quad + 2\left[\xi^T(t)I_0^TY(p_{mj,t}^o, p_{mj,t}) + x^T(t-\tau_{p_{mj,t}}(t))T(p_{mj,t}^o, p_{mj,t})\right]\\ &\quad \times\left[I_0\xi(t) - \int_{t-\tau_{p_{mj,t}}(t)}^t \dot{x}(s)\mathrm{d}s - x(t-\tau_{p_{mj,t}}(t))\right]\\ &\quad + \mu\zeta^T(t)X(p_{mj,t}^o, p_{mj,t})\zeta(t)\\ &\quad - \int_{t-\tau_{p_{mj,t}}(t)}^t \zeta^T(t)X(p_{mj,t}^o, p_{mj,t})\zeta(t)\,\mathrm{d}s\\ &= \zeta^T(t)\Xi_{ii}\zeta(t) - \int_{t-\tau_i(t)}^t \chi^T(t,s)\Gamma_{ii}\chi(t,s)\,\mathrm{d}s, \qquad (28)\end{aligned}$$

where

$$\chi^T(t,s) = \left[\xi^T(t)\ \ x^T(t-\tau_i(t))\ \ \dot{x}^T(s)\right]$$

$$\Xi_{ii} = \begin{bmatrix}\hat{\Phi}_{11} + \mu\hat{A}_{1ii}^TI_0^TZI_0\hat{A}_{1ii} & \hat{\Phi}_{12} + \mu\hat{A}_{1ii}^TI_0^TZI_0\hat{A}_{2i}\\ \hat{\Phi}_{12}^T + \mu\hat{A}_{2i}^TI_0^TZI_0\hat{A}_{1ii} & \hat{\Phi}_{22} + \hat{A}_{2i}^TI_0^TZI_0\hat{A}_{2i}\end{bmatrix}$$

$$\hat{\Phi}_{11} = \begin{cases}\hat{A}_{1ii}^TP_{ii} + P_{ii}\hat{A}_{1ii} + \sum_{j=1}^N q_{ij}P_{ij}\\ \quad + \sum_{j=1}^N q_{ij}^1P_{ji} + I_0^TY_{ii}I_0 + I_0Y_{ii}^TI_0^T + (1+\eta\mu)I_0^TQI_0\\ \quad + \mu X_{1ii}, \quad \text{if } j = i\\ \hat{A}_{1ji}^TP_{ji} + P_{ji}\hat{A}_{1ji} + q_{ji}^0(P_{ii} - P_{ji}) + I_0^TY_{ji}I_0\\ \quad + I_0Y_{ji}^TI_0^T + (1+\eta\mu)I_0^TQI_0 + \mu X_{1ji}, \quad \text{if } j \neq i\end{cases}$$

$$\hat{\Phi}_{12} = P_{ji}\hat{A}_{2i} - I_0^TY_{ji} + I_0^TT_{ji}^T + \mu X_{2ji},$$

$$\hat{\Phi}_{22} = -T_{ji} - T_{ji}^T - (1-h_i)Q + \mu X_{3ji}$$

$$\eta = \max_{i\in S}\{|q_{ii}|\}.$$

If $\Xi_{ii} < 0$, $\Gamma_{ii} \geq 0$, then for each $i \in S$ and any scalar $\beta > 0$, we have

$$\mathscr{L}\left[e^{\beta t}V(\xi_t, i, i, t)\right] \leq -\alpha_1e^{\beta t}\|\xi(t)\|^2 + \beta e^{\beta t}V(\xi_t, i, i, t), \quad \forall i\in, \beta > 0 \quad (29)$$

where $\alpha_1 = \min_{i\in S}\{\lambda_{\min}(-\Xi_{ii})\}$. Similar to Xu *et al.* (2003), we can verify that

$$V(\xi_t, i, i, t) \le \lambda_{\max}(P_{ii})\|\xi(t)\|^2 + \lambda_{\max}(Q)\int_{t-\tau_i(t)}^{t} \|x(s)\|^2 \mathrm{d}s + \eta\lambda_{\max}(Q)\int_{-\mu}^{0}\int_{t+\theta}^{t}\|x(s)\|^2\mathrm{d}s\,\mathrm{d}\theta + \lambda_{\max}(Z)\int_{-\mu}^{0}\int_{t+\theta}^{t}\|\dot{x}(s)\|^2\,\mathrm{d}s\,\mathrm{d}\theta$$
$$\le \lambda_{\max}(P_{ii})\|\xi(t)\|^2 + (\eta\mu+1)\lambda_{\max}(Q)\int_{t-\mu}^{t}\|x(s)\|^2\mathrm{d}s + \mu\lambda_{\max}(Z)\int_{t-\mu}^{t}\|\dot{x}(s)\|^2\mathrm{d}s.$$

Noticing that in nominal system Σ_0

$$\dot{\xi}(t) = \hat{A}_{1ii}\xi(t) + \hat{A}_{2i}I_0\xi(t-\tau_{p_{mj,t}}(t))$$

and letting $\alpha_2 = \max_{i\in S}\{2\|\hat{A}_{1ii}\|^2\}$, $\alpha_3 = \max_{i\in S}\times\{2\|\hat{A}_{2i}\|^2\}$, we have

$$\|\dot{\xi}(t)\|^2 \le \alpha_2\|\xi(t)\|^2 + \alpha_3\|x(t-\tau_{p_{mj,t}}(t))\|^2.$$

This, together with (29), gives

$$\mathscr{L}[e^{\beta t}V(\xi_t, i, i, t)] \le (-\alpha_1+\alpha_4\beta)e^{\beta t}\|\xi(t)\|^2 + \alpha_3\mu\lambda_{\max}(Z)\beta e^{\beta t}\int_{t-\mu}^{t}\|x(s-\tau p_{mj,s}(t))\|^2\mathrm{d}s + \beta e^{\beta t}[(\mu\eta+1)\lambda_{\max}(Q) + \alpha_2\mu\lambda_{\max}(Z)]\int_{t-\mu}^{t}\|x(s)\|^2\mathrm{d}s \tag{30}$$

where $\alpha_4 = \max_{i\in S}\{\lambda_{\max}(P_{ii})\}$.

Using Dynkin's formula (let $f\in C^2$, then $E[f(x_T)|x] - f(x) = E[\int_0^T \mathscr{L}f(x_s)\mathrm{d}s|x]$) (Kushner 1967), for any $T>0$, $\beta>0$, and each $p^o_{mj,t} = p_{mj,t} = i$, $i\in S$, we have

$$E\left\{e^{\beta T}V(\xi_T, p^o_{mj,T}, p_{mj,T}, T)|\xi_0, p^o_{mj,0}, p_{mj,0}, 0\right\} = V(\xi_0, p^o_{mj,0}, p_{mj,0}, 0) + E\left\{\int_0^T \mathscr{L}\left[e^{\beta s}V(\xi_s, i, i, s)\right]\mathrm{d}s|\xi_0, p^o_{mj,0}, p_{mj,0}, 0\right\}.$$

Since the initial time values $x(0)=x_0, p_{mj,0}$ and $p^o_{mj,0}$ are deterministic, that means ξ_0 is deterministic too. Taking (30) into it gives

$$E\left\{e^{\beta T}V(\xi_T, p^o_{mj,T}, p_{mj,T}, T\right\} \le V(\xi_0, p^o_{mj,0}, p_{mj,0}, 0) + E\left\{(-\alpha_1+\alpha_4\beta)\int_0^T e^{\beta t}\|\xi(t)\|^2\mathrm{d}t + \beta[(\mu\eta+1)\lambda_{\max}(Q) + \alpha_2\mu\lambda_{\max}(Z)]\int_0^T e^{\beta t}\int_{t-\mu}^{t}\|x(s)\|^2\mathrm{d}s\,\mathrm{d}t + \alpha_3\mu\lambda_{\max}(Z)\beta\int_0^T e^{\beta t}\int_{t-\mu}^{t}\|x(s-\tau p_{mj,s}(t))\|^2\mathrm{d}s\,\mathrm{d}t\right\} \tag{31}$$

Letting $\bar{\theta} = t - \tau_i(t)$ and observing the following inequalities

$$\begin{cases} \dot{\tau}_i(t) = \dfrac{\mathrm{d}\tau_i(t)}{\mathrm{d}t} \le h_i < 1 \\ \mathrm{d}t \le \dfrac{1}{1-h_i}\,\mathrm{d}\bar{\theta} \end{cases} \tag{32}$$

we have

$$\int_0^T e^{\beta t}\int_{t-\mu}^{t}\|x(s)\|^2\mathrm{d}s\,\mathrm{d}t \le \int_{-\mu}^{0}\mu e^{\beta(s+\mu)}\|x(s)\|^2\mathrm{d}s + \int_0^{T-\mu}\mu e^{\beta(s+\mu)}\|x(s)\|^2\mathrm{d}s + \int_{T-\mu}^{T}\mu e^{\beta(s+\mu)}\|x(s)\|^2\,\mathrm{d}s$$
$$= \mu\int_{-\mu}^{T} e^{\beta(t+\mu)}\|x(t)\|^2\mathrm{d}t \le \mu\int_{-\mu}^{T} e^{\beta(t+\mu)}\|\xi(t)\|^2\,\mathrm{d}t, \tag{33}$$

$$\int_0^T e^{\beta t}\int_{t-\mu}^{t}\|x(s-\tau p_{mj,s}(t)\|^2\,\mathrm{d}s\,\mathrm{d}t \le \int_{-\mu}^{0}\mu e^{\beta(s+\mu)}\|x(s-\tau p_{mj,s}(t))\|^2\mathrm{d}s + \int_0^{T-\mu}\mu e^{\beta(s+\mu)}\|x(s-\tau p_{mj,s}(t))\|^2\mathrm{d}s + \int_{T-\mu}^{T}\mu e^{\beta(s+\mu)}\|x(s-\tau p_{mj,s}(t))\|^2\,\mathrm{d}s$$
$$= \mu\int_{-\mu}^{T} e^{\beta(t+\mu)}\|x(t-\tau_{Pmj,t}(t))\|^2\mathrm{d}t \le \frac{1}{1-h_i}\mu\int_{-2\mu}^{T} e^{\beta(\bar{\theta}+2\mu)}\|x(\bar{\theta})\|^2\mathrm{d}\bar{\theta} = \frac{1}{1-h_i}\mu\int_{-2\mu}^{T} e^{\beta(t+2\mu)}\|x(t)\|^2\mathrm{d}t \le \frac{1}{1-h_i}\mu\int_{-2\mu}^{T} e^{\beta(t+2\mu)}\|\xi(t)\|^2\mathrm{d}t. \tag{34}$$

Substituting (33) and (34) into (31) leads to

$$\begin{aligned}
&E\Big\{e^{\beta T}V(\xi_T, p^o_{mj,T}, p^o_{mj,T}, T)\Big\}\\
&\le V(\xi_0, p^o_{mj,0}, p_{mj,0}, 0)\\
&\quad + E\Big\{(-\alpha_1+\alpha_4\beta)\int_0^T e^{\beta t}\|\xi(t)\|^2\,\mathrm{d}t\\
&\quad + \beta[(\mu\eta+1)\lambda_{\max}(Q)+\alpha_2\mu\lambda_{\max}(Z)]\mu\\
&\quad \times\int_{-\mu}^T e^{\beta(t+\mu)}\|\xi(t)\|^2\mathrm{d}t\\
&\quad + \frac{\alpha_3\mu^2\lambda_{\max}(Z)\beta}{1-h_i}\int_{-2\mu}^T e^{\beta(t+2\mu)}\|\xi(t)\|^2\,\mathrm{d}t\Big\}\\
&\le V(\xi_0, p^o_{mj,0}, p_{mj,0}, 0)\\
&\quad + E\Big\{\alpha_5\beta e^{\beta\mu}\int_{-\mu}^0\|\xi(t)\|^2\,\mathrm{d}t + \alpha_6\beta e^{2\beta\mu}\int_{-2\mu}^0\|\xi(t)\|^2\,\mathrm{d}t\\
&\quad + \big[-\alpha_1+\alpha_4\beta+\alpha_5\beta e^{\beta\mu}+\alpha_6\beta e^{2\beta\mu}\big]\int_0^T e^{\beta t}\|\xi(t)\|^2\mathrm{d}t\Big\}
\end{aligned}$$

where $\alpha_5 = [(\mu\eta+1)\lambda_{\max}(Q)+\alpha_2\mu\lambda_{\max}(Z)]\mu$, and $\alpha_6 = (\alpha_3\mu^2\lambda_{\max}(Z)/(1-h_i))$.

Choose $\beta > 0$ such that

$$-\alpha_1+\alpha_4\beta+\alpha_5\beta e^{\beta\mu}+\alpha_6\beta e^{2\beta\mu}\le 0.$$

Then, we have

$$E\{e^{\beta T}V(\xi_T, p^o_{mj,T}, p_{mj,T})\}\le c, \tag{35}$$

where $c = V(\xi_0, p^o_{mj,0}, p_{mj,0}, 0) + E\{\alpha_5\beta e^{\beta\mu}\int_{-\mu}^0\|\xi(t)\|^2\mathrm{d}t + \alpha_6\beta e^{2\beta\mu}\int_{-2\mu}^0\|\xi(t)\|^2\mathrm{d}t\}$.

Hence, the LMIs $\Xi_{ii}<0$, $\Gamma_{ii}\ge 0$ guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square, for $p^o_{mj,t} = p_{mj,t} = i$, $\forall i\in S$.

Case 2: $p^o_{mj,t} = j$, $p_{mj,t} = i$, and $j\ne i$. Following similar lines as in the proof of the Case 1, we have

$$\begin{aligned}
\mathscr{L}V(x_t, j, i, t) &\le \xi^T(t)\Big[\hat{A}^T_{1ji}P_{ji}+P_{ji}\hat{A}_{1ji}+q^0_{ji}(P_{ii}-P_{ji})\Big]\xi(t)\\
&\quad + \xi^T(t)P_{ji}\hat{A}_{2i}x(t-\tau_i(t))\\
&\quad + x^T(t-\tau_i(t))\hat{A}^T_{2i}P_{ji}\xi(t)\\
&\quad + (1+\eta\mu)\xi^T(t)I_0^TQI_0\xi(t)\\
&\quad - (1-h_i)x^T(t-\tau_i(t))Qx(t-\tau_i(t))\\
&\quad - \mu\dot{\xi}^T(t)I_0^TZI_0\dot{\xi}(t) - \int_{t-\tau_i(t)}^t \dot{x}^T(s)Z\dot{x}(s)\,\mathrm{d}s\\
&\le \xi^T(t)\Xi_{ji}\xi(t) - \int_{t-\tau_i(t)}^t \chi^T(t,s)\Gamma_{ji}\chi(t,s)\,\mathrm{d}s,
\end{aligned} \tag{36}$$

where

$$\Xi_{ji} = \begin{bmatrix} \hat{\Phi}_{11}+\mu\hat{A}^T_{1ji}I_0^TZI_0\hat{A}_{1ji} & \hat{\Phi}_{12}+\mu\hat{A}^T_{1ji}I_0^TZI_0\hat{A}_{2i}\\ \hat{\Phi}^T_{12}+\mu\hat{A}^T_{2i}I_0^TZI_0\hat{A}_{1ji} & \hat{\Phi}_{22}+\hat{A}^T_{2i}I_0^TZI_0\hat{A}_{2i}\end{bmatrix}$$

and the LMIs $\Xi_{ji}<0$, $\Gamma_{ji}\ge 0$ guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square, for $p^o_{mj,t}=j$, $p_{mj,t}=i$ and $j\ne i$, $\forall j, i\in S$.

Applying the Schur complement, it can be shown that for any $i, j\in S$, $\Xi_{ji}<0$ implies

$$\begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & \mu\hat{A}^T_{1ji}I_0^TZ\\ \hat{\Phi}^T_{12} & \hat{\Phi}_{22} & \mu\hat{A}^T_{2i}I_0^TZ\\ \mu ZI_0\hat{A}_{1ji} & \mu ZI_0\hat{A}_{2i} & -\mu Z\end{bmatrix} < 0 \tag{37}$$

which is equivalent to the following condition:

$$\begin{aligned}
&\begin{bmatrix} \Phi_{11} & C_i^TB_{3j}^TP_{2ji}+\mu X^2_{1ii} & \Phi_{13} & \mu A^T_{1i}Z\\ P_{2ji}B_{3j}C_i+\mu X^{2^T}_{1ii} & A^T_{3j}P_{2ji}+P_{2ji}A_{3j}+\Phi_{22} & \mu X^2_{2ii} & 0\\ \Phi^T_{13} & \mu X^{2^T}_{2ii} & \Phi_{33} & \mu A^T_{2i}Z\\ \mu ZA_{1i} & 0 & \mu ZA_{2i} & -\mu Z\end{bmatrix}\\
&+\begin{bmatrix} P_{1ji}B_{1i}\\ 0\\ 0\\ \mu ZB_{1i}\end{bmatrix} I\big[0\ \ K_j\ \ 0\ \ 0\big] + \begin{bmatrix} 0\\ K_j^T\\ 0\\ 0\end{bmatrix} I\big[B^T_{1i}P_{1ji}\ \ 0\ \ 0\ \ \mu B^T_{1i}Z\big] < 0.
\end{aligned} \tag{38}$$

By Lemma 2, a sufficient condition guaranteeing (38) is that there exists a positive number $\rho_{ji}>0$ such that

$$\begin{aligned}
&\rho_{ji}\begin{bmatrix} \Phi_{11} & C_i^TB_{3j}^TP_{2ji}+\mu X^2_{1ii} & \Phi_{13} & \mu A^T_{1i}Z\\ P_{2ji}B_{3j}C_i+\mu X^{2^T}_{1ii} & A^T_{3j}P_{2ji}+P_{2ji}A_{3j}+\Phi_{22} & \mu X^2_{2ii} & 0\\ \Phi^T_{13} & \mu X^{2^T}_{2ii} & \Phi_{33} & \mu A^T_{2i}Z\\ \mu ZA_{1i} & 0 & \mu ZA_{2i} & -\mu Z\end{bmatrix}\\
&+\rho^2_{ji}\begin{bmatrix} P_{1ji}B_{1i}\\ 0\\ 0\\ \mu ZB_{1i}\end{bmatrix} I\big[B^T_{1i}P_{1ji}\ \ 0\ \ 0\ \ \mu B^T_{1i}Z\big] + \begin{bmatrix} 0\\ K_i^T\\ 0\\ 0\end{bmatrix} I\big[0\ \ K_i\ \ 0\ \ 0\big] < 0.
\end{aligned} \tag{39}$$

Replacing $\rho_{ji}P_{1ji}$, $\rho_{ji}P_{2ji}$, $\rho_{ji}Q$, $\rho_{ji}Z$, $\rho_{ji}X_{ji}$, $\rho_{ji}Y_{ji}$ and $\rho_{ji}T_{ji}$ with P_{1ji}, P_{2ji}, Q, Z, X_{ji}, Y_{ji} and T_{ji}, respectively, and applying the Schur complement shows that (39) is equivalent to

$$\mathbf{W}_{ji} = \begin{bmatrix} \Phi_{11} & C_i^T V_{ji} + \mu X_{1ii}^2 & \Phi_{13} & \mu A_{1i}^T Z & P_{1ji} B_{1i} & 0 \\ V_{ji}^T C_i + \mu X_{1ii}^{2^T} & U_{ji} + U_{ji}^T + \Phi_{22} & \mu X_{2ii}^2 & 0 & 0 & K_j^T \\ \Phi_{13}^T & \mu X_{2ii}^{2^T} & \Phi_{33} & \mu A_{2i}^T Z & 0 & 0 \\ \mu Z A_{1i} & 0 & \mu Z A_{2i} & -\mu Z & \mu Z B_{1i} & 0 \\ B_{1i}^T P_{1ji} & 0 & 0 & \mu B_{1i}^T Z & -I & 0 \\ 0 & K_j & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad j, i \in S. \tag{40}$$

Hence, the LMIs (15), (40) guarantee that the nominal time-delayed jump linear system Σ_0 is exponentially stable in mean square for $p^o_{mj,t} = j$, $p_{mj,t} = i$, $\forall j, i \in S$.

Then, for the uncertain time-delayed jump linear system (11) without disturbance, replacing A_{1i}, A_{2i}, B_{1i} and K_j in (40) with $A_{1i} + H_{1i}F_i(t)E_{1i}$, $A_{2i} + H_{1i}F_i(t)E_{2i}$, $B_{1i} + H_{1i}F_i(t)E_{3i}$ and $K_j + \alpha_i\phi_i(t)K_j$, we can obtain that (40) for system (11) is equivalent to the following condition:

$$\mathbf{W}_{ji} + \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} P_{ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix} \phi_i(t) \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \phi_i^T(t) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix}^T < 0. \tag{41}$$

By Lemma 3.2, a sufficient condition guaranteeing (41) is that there exist positive numbers $\rho_{1ji} > 0$, $\rho_{2ji} > 0$, $\rho_{3ji} > 0$ such that

$$\mathbf{W}_{ji} + \rho_{3ji}^{-1} \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P_{1ji}H_{1i} \\ 0 \\ 0 \\ \mu Z H_{1i} \\ 0 \\ 0 \end{bmatrix}^T + \rho_{3ji} \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{1i}^T \\ 0 \\ E_{2i}^T \\ 0 \\ E_{3i}^T \\ 0 \end{bmatrix}^T + \rho_{2ji}^{-1} \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ V_{ji}^T H_{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \rho_{2ji} \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_{4i}^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \rho_{1ji} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_i \end{bmatrix}^T + \rho_{1ji}^{-1} \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ K_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{42}$$

Applying the Schur complement shows that (14) is equivalent to (42) for all $p^o_{mj,t} = j$, $p_{mj,t} = i$, $\forall j, i \in S$. This completes the proof. □

Appendix B. Proof of Theorem 2

Proof: For the nominal time-delayed jump linear system Σ_1 with disturbance

$$\Sigma_1: \begin{cases} \dot{\xi}(t) = \hat{A}_1(p^o_{mj,t}, p_{mj,t})\xi(t) + \hat{A}_2(p_{mj,t})I_0\xi(t - \tau_{p_{mj,t}}(t)) \\ \qquad + \hat{B}_2(p_{mj,t})w(t) \\ z(t) = \left[C(p_{mj,t}) + \Delta_C(p_{mj,t}, t)\right] I_0\xi(t) \\ I_0\xi(s) = f(s), \quad p_{mj,s} = p_{mj,0}, \quad s \in [-\mu, 0], \end{cases} \tag{43}$$

where

$$\hat{B}_2(p_{mj,t}) = \begin{bmatrix} B_2(p_{mj,t}) \\ 0 \end{bmatrix}.$$

Letting $\bar{\zeta}^T(t) = [\xi^T(t)\ x^T(t-\tau_i(t))\ w^T(t)]$, taking the Lyapunov function candidate as (24), and employing the following conditions

$$\mu\bar{\zeta}^T(t)\bar{X}(p^o_{mj,t}, p_{mj,t})\bar{\zeta}(t) - \int_{t-\tau_{p_{mj,t}}(t)}^{t} \bar{\zeta}^T(t)\bar{X}(p^o_{mj,t}, p_{mj,t})\bar{\zeta}(t)\,\mathrm{d}s \geq 0,$$

$$2\Big[\xi^T(t)I_0^T Y(p^o_{mj,t}, p_{mj,t}) + x^T(t-\tau_{p_{mj,t}}(t))T(p^o_{mj,t}, p_{mj,t}) + w^T(t)N(p^o_{mj,t}, p_{mj,t})\Big] \times \left[I_0\xi - \int_{t-\tau_{p_{mj,t}}(t)}^{t} \dot{x}(s)\,\mathrm{d}s - x(t-\tau_{p_{mj,t}}(t))\right] = 0 \qquad (44)$$

we can obtain

$$\mathscr{L}V(x_t, j, i, t) \leq \bar{\zeta}^T(t)\bar{\Xi}_{ji}\bar{\zeta}(t) - \int_{t-\tau_i(t)}^{t} \bar{\chi}^T(t,s)\bar{\Gamma}_{ji}\bar{\chi}(t,s)\,\mathrm{d}s, \qquad (45)$$

where

Using Dynkin's formula again (Kushner 1967), we have

$$E\left\{\int_0^T \mathscr{L}V(x_s, p^o_{mj,s}, p_{mj,s}, s)\mathrm{d}s\right\} = E\Big\{V(x_T, p^o_{mj,T}, p_{mj,T}, T)\Big\} - E\Big\{V(x_0, p^o_{mj,0}, p_{mj,0}, 0)\Big\}.$$

Under the zero initial condition ($x(0) = 0$), we have

$$E\Big\{V(x_0, p^o_{mj,0}, p_{mj,0}, 0)\Big\} = 0.$$

Thus, for any $w(t) \in L_2[0\quad \infty)$, we have

$$\begin{aligned} J &= E\left\{\int_0^T \Big[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathscr{L}V(x_t, p^o_{mj,t}, p_{mj,t}, t)\Big]\mathrm{d}t\right\} \\ &\quad - E\Big\{V(x_T, p^o_{mj,T}, p_{mj,T}, T)\Big\} \\ &\leq E\left\{\int_0^T \Big[z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \mathscr{L}V(x_t, p^o_{mj,t}, p_{mj,t}, t)\Big]\mathrm{d}t\right\}. \end{aligned} \qquad (46)$$

$$\bar{\Xi}_{ji} = \begin{bmatrix} \bar{\Psi}_{11} + \mu\hat{A}_{1ji}^T I_0^T Z I_0 \hat{A}_{1ji} & \bar{\Psi}_{12} + \mu\hat{A}_{1ji}^T I_0^T Z I_0 \hat{A}_{2i} & \bar{\Psi}_{13} + \hat{A}_{1ji}^T I_0^T Z I_0 \hat{B}_{2i} \\ \bar{\Psi}_{12}^T + \mu\hat{A}_{2i}^T I_0^T Z I_0 \hat{A}_{1ji} & \bar{\Psi}_{22} + \mu\hat{A}_{2i}^T I_0^T Z I_0 \hat{A}_{2i} & \bar{\Psi}_{23} + \mu\hat{A}_{2i}^T I_0^T Z I_0 \hat{B}_{2i} \\ \bar{\Psi}_{13}^T + \mu\hat{B}_{2i}^T I_0^T Z I_0 \hat{A}_{1ji} & \bar{\Psi}_{23}^T + \mu\hat{B}_{2i}^T I_0^T Z I_0 \hat{A}_{2i} & \bar{\Psi}_{33} + \mu\hat{B}_{2i}^T I_0^T Z I_0 \hat{B}_{2i} \end{bmatrix}$$

$$\bar{\Psi}_{11} = \begin{cases} \hat{A}_{1ii}^T P_{ii} + P_{ii}\hat{A}_{1ii} + I_0^T Y_{ii} I_0 + I_0 Y_{ii}^T I_0^T + (1+\eta\mu)I_0^T Q I_0 \\ \quad + \mu\bar{X}_{11ii} + \sum_{j=1}^{N} q_{ij}P_{ij} + \sum_{j=1}^{N} q_{ij}^1 P_{ji}, \quad \text{if } j = i \\ \hat{A}_{1ji}^T P_{ji} + P_{ji}\hat{A}_{1ji} + I_0^T Y_{ji} I_0 + I_0 Y_{ji}^T I_0^T + (1+\eta\mu)I_0^T Q I_0 + \mu\bar{X}_{11ji} \\ \quad + q_{ji}^0(P_{ii} - P_{ji}), \quad \text{if } j \neq i \end{cases}$$

$$\bar{\Psi}_{12} = P_{ji}\hat{A}_{2i} - I_0^T Y_{ji} + I_0^T T_{ji}^T + \mu\bar{X}_{12ji},$$

$$\bar{\Psi}_{22} = -T_{ji} - T_{ji}^T - (1-h_i)Q + \mu\bar{X}_{22ji},$$

$$\bar{\Psi}_{13} = P_{ji}\hat{B}_{2i} + I_0^T N_{ji}^T + \mu\bar{X}_{13ji},$$

$$\bar{\Psi}_{23} = -N_{ji}^T + \mu\bar{X}_{23ji}, \quad \bar{\Psi}_{33} = \bar{X}_{33ji}, \quad \eta = \max_{i\in S}\{|q_{ii}|\}$$

$$\bar{\chi}^T(t,s) = \big[\xi^T(t)\quad x^T(t-\tau_i(t))\quad w^T(t)\quad \dot{x}^T(s)\big].$$

Taking (45) into the above inequality gives

$$J \le E\left\{\int_0^T\left[\bar{\zeta}^T(t)\left(\bar{\Xi}_{ji}+\begin{bmatrix} I_0^T C_i^T C_i I_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma^2 I\end{bmatrix}\right)\bar{\zeta}(t) - \int_{t-\tau_i(t)}^{t}\bar{\chi}^T(t,s)\bar{\Gamma}_{ji}\bar{\chi}(t,s)\,\mathrm{d}s\right]\mathrm{d}t\right\}.$$

By Lemma 2 and applying the Schur complement shows that (19) and

$$\begin{bmatrix} \Psi_{11}+\rho_{4ji}^{-1}C_i^T C_i & C_i^T B_{3j}^T \hat{P}_{2ji}+\mu\hat{X}_{11ji}^2 & \Psi_{13} & \Psi_{14} & \mu A_{1i}^T\hat{Z} & \hat{P}_{1ji}B_{1i} & 0 \\ \hat{P}_{2ji}B_{3j}C_i+\mu\hat{X}_{11ji}^{2^T} & A_{3j}^T\hat{P}_{2ji}+\hat{P}_{2ji}A_{3j}+\Psi_{22} & \mu\hat{X}_{12ji}^2 & \mu\hat{X}_{13ji}^2 & 0 & 0 & K_j^T \\ \Psi_{13}^T & \mu\hat{X}_{12ji}^{2^T} & \Psi_{33} & \Psi_{34} & \mu A_{2i}^T\hat{Z} & 0 & 0 \\ \Psi_{14}^T & \mu\hat{X}_{13ji}^{2^T} & \Psi_{34}^T & \Psi_{44}-\gamma^2 I & \mu B_{2i}^T\hat{Z} & 0 & 0 \\ \mu\hat{Z}A_{1i} & 0 & \mu\hat{Z}A_{2i} & \mu\hat{Z}B_{2i} & -\mu\hat{Z} & \hat{Z}B_{1i} & 0 \\ B_{1i}^T\hat{P}_{1ji} & 0 & 0 & 0 & \mu B_{1i}^T\hat{Z} & -I & 0 \\ 0 & K_j & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \qquad (47)$$

guarantee $J < 0$ for any $w(t) \neq 0$ (and $w(t) \in L_2[0,\infty)$), which also guarantee γ-disturbance H_∞ attenuation (12) of the closed-loop system Σ_1 from $w(t)$ to $z(t)$.

Then, replacing A_{1i}, A_{2i}, B_{1i}, C_i and K_j in (47) with $A_{1i}+H_{1i}F_{i(t)}E_{1i}$, $A_{2i}+H_{1i}F_{i(t)}E_{2i}$, $B_{1i}+H_{1i}F_i(t)E_{3i}$, $C_i+H_{2i}F_i(t)E_{4i}$, and $K_j+\alpha_i\Phi_i(t)K_j$ and using the similar proof of Theorem 1, we can easily verify that the control $u(t)=K(p^o_{mj,t})x(t)$ guarantees γ-disturbance H_∞ attenuation (12) of the closed-loop system (11) from $w(t)$ to $z(t)$, if the coupled linear matrix inequalities (18) and (19) satisfied. This completes the proof. □

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